RR-0255

M. A. / M. Sc. (Mathematics) (Part - I) (External)
Examination
March / April - 2017
405 : Graph Theory & Discrete Structure

Time : Hours] [Total Marks : 100

Instructions :
(1) Fill up strictly the details of signs on your answer book.
(2) Attempt all questions.
(3) Figures to the right indicate marks.
(4) Follow the usual notations and conventions.

1. Attempt any FIVE.

1. Show that an infinite graph with a finite number of vertices will have at least one pair of vertices joined by an infinite number of parallel edges.
2. Prove that every self-loop is a circuit but converse is not true.
3. In a graph G, let P1 and P2 be two different paths between two given vertices. Prove that P1 ⊕ P2 is a circuit or a set of circuits in G.
4. Prove that every connected graph has at least one spanning tree.
5. Prove that every finite lattice is complete.
7. Discuss clock algebra.

2(a) Prove that any connected graph with n vertices and n - 1 edges is a tree.
(b) Prove that a given connected graph G is an Euler graph iff all vertices of G are of even degree.
(c) What is the maximum possible height of an n - vertex binary tree?

OR

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2(a) Prove that a tree with \( n \) vertices has \( n - 1 \) edges.

(b) Sketch all the spanning tree of the following graph. Also for one of them, show that rank and nullity theorem holds.

(c) Show that the distance between vertices of a connected graph is a metric.

3(a) Prove that in a connected graph \( G \), a vertex \( v \) is a cut-vertex iff there exist at least two edges \( x \) and \( y \) incident on \( v \) such that no circuit in \( G \) includes both \( x \) and \( y \).

(b) Prove that the complete graph of five vertices is non-planar.

(c) Prove that an Euler graph \( G \) is arbitrarily traceable from vertex \( v \) in \( G \) iff every circuit in \( G \) contains \( v \).

OR

3(a) If \( A(G) \) is an incidence matrix of a connected graph \( G \) with \( n \) vertices. Prove that the rank of \( A(G) \) is \( n - 1 \).

(b) Prove that a chord \( c_i \), that determines a fundamental cut-circuit \( R \), is contained in every fundamental cut-set associated with the branch of \( R \) and in no other.

(c) Prove that a graph can be embedded in a surface of the sphere iff it can be embedded in a plane.

4(a) Prove that \( < Z_m, +_m > \) and \( < Z_m, \cdot_m > \) are isomorphic.

(b) Prove that the direct product of two commutative semigroups is a commutative semigroup.

(c) Let \( S = \{ p, q, r \} \) then prove that \( < p(S), \cup > \) and \( < p(S), \cap > \) are monoids.

OR

4(a) What are the disadvantages of residue number system?

(b) Prove that the rank of a well-formed polish notation is 1.

(c) Prove that if \( a \) and \( m \) are relatively prime integers then

\[
a' = a^{(m)-1} \mod m.
\]
5(a) Prove that the direct product of two distributive lattices is a distributive lattice.

(b) Show that in a complemented distributive lattice,
\[ a \leq b \iff a \ast b' = 0 \iff a' \Theta b = 1 \iff b' \leq a'. \]

(c) Prove that \( \bigoplus_{i=0}^{n-1} \min_i = 1 \) for three variables.

OR

5(a) For a distributive lattice \(< L, \ast, \Theta >\), prove that \((a \ast b = a \ast c) \land (a \Theta b = a \Theta c) \Rightarrow b = c; a, b, c \in L.\)

(b) Minimize the following Boolean function:
\[ f(x, y, z) = x'y'z + x'y + x. \]

(c) Represent the following Boolean function using circuit diagram and n-space representation:
\[ f(x, y, z) = \Sigma(0,2,6,7). \]