RAN-1188

RAN-1188

B.Sc (Sem.-V) Examination

March / April - 2019

Mathematics : Paper -606

(Number Theory)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book
Name of the Examination:
☛ **B.Sc (Sem.-V)**
Name of the Subject :
☛ **Mathematics : Paper -606 (Number Theory)**
Subject Code No.: 1 1 8 8

Seat No.:

Student's Signature

(2) Digits to the right indicates marks of the question.

(3) Follow the usual notations.

Q-1 Answer any five questions : (10)

(1) Show that 561 is a Pseudo prime .

(2) Arrange the integers 2, 3, 4, ------ , 21 in pairs a and b such that $ab \equiv 1 \ (mod\ 23)$.

(3) Show that $a^{21} \equiv a$ (mod 15) for any integer a.

(4) Prove that $\sum_{k=1}^{n} \mu(k!) = 1$ where $n \geq 3$.

(5) Show that $\tau(m + 1) = \tau(m + 2)$ for $m = 3655$.

(6) Solve : $36x \equiv 8(mod\ 102)$.

(7) Show that $[x] + [-x] = 0$ or -1 according as x is an integer or not

(8) Prove that $\phi(n) + 2 = \phi(n + 2)$ for $n = 4p$ where p and $2p + 1$ both are odd primes.

Q-2 Answer any two questions : (10)

(1) State and prove Chinese Remainder Theorem .

(2) Solve the linear congruence $34x \equiv 60(mod\ 98)$.

(3) Obtain three consecutive integers, the first of which is divisible by a fourth Power, the second by a cube and third by a square.

Q-3 Answer any two questions : (10)

(1) If n is an odd pseudo prime then prove that $M_n = 2^n - 1$ is larger one.

(2) If p and q are distinct primes with $a^p \equiv a\ (mod\ q)$ and $a^q \equiv a\ (mod\ p)$ then prove that $a^{pq} \equiv a$ (mod pq).

(3) If p is a prime then for any integer a prove that

(i) $p \mid a^p + (p-1)!a$ (ii) $p \mid (p-1)!a^p + a$

Q-4 Answer any two questions : (10)

(1) Prove that τ and σ functions are multiplicative .

(2) For any positive integer n; show that $\mu(n).\mu(n+1).\mu(n+2).\mu(n+3) = 0$

(3) Find the highest power of 65 which divides 1000!

Q-5 Answer any two questions : (10)

(1) If m and n are relatively prime positive integer then prove that $m^{\phi(n)} + n^{\phi(m)} \equiv 1\ (\text{mod}\ mn)$.

(2) Prove that $\varphi(n) = \frac{n}{2}$ iff $n = 2^k$ for some $k \geq 1$.

(3) Prove that $\varphi(2n) = \varphi(n)$ or $\varphi(2n) = 2\varphi(n)$ according as n is an odd Or even integer.