RAN-1186

B. Sc. Sem -VI Examination

March / April - 2019

Mathematics Paper : MTH - 604

Real Analysis - IV

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **B. Sc. Sem -VI**

Name of the Subject :

☛ **Mathematics Paper : MTH - 604**

Subject Code No.: 1 1 8 6

Seat No.:

Student's Signature

(1) All questions are compulsory.
(2) Figures to the right indicate marks of corresponding question.
(3) Follow usual notations.
(4) Use of non-programmable scientific calculator is allowed.

1. Answer the following as directed : (Any FIVE) **(10)**

(1) Prove that every finite set in any metric space is closed.

(2) Justify: (2019,2020) is a closed subset of the metric space $\langle (2019,2020), |.| \rangle$.

(3) Justify: $[0,1] \cup [2,3]$ is a connected set of the metric space R^1.

(4) Prove that $(0, \infty)$ is a bounded subset of the metric space R_d and its diameter is 1.

(5) Justify : R_d is not the complete metric space.

(6) State Picard's Fixed - Point Theorem.

(7) (i) Give an example of a compact subset of R^1 which is not connected;

(ii) Give an example of a subset of R_d which is compact as well as connected.

(8) State Finite - Intersection property and give its illustration in the metric space R^1 .

2. Attempt any TWO : (10)

(1) Let E be the subset of a metric space $\langle M, \rho \rangle$. Prove that $\overline{E}$; the closure of E ; is closed.

(2) Define a closed set in a metric space. Prove that a finite intersection of closed sets in any metric space is closed.

(3) If A and B are closed subsets of R^1, then prove that A x B is a closed subset of R^2 .

3. Attempt any TWO : (10)

(1) If the metric space M is connected, then prove that every continuous characteristic function on M is constant.

(2) If A is a connected subset of a metric space $\langle M, \rho \rangle$, then prove that $\overline{A}$ is also connected.

(3) Define a totally bounded set. If A is a totally bounded subset of the metric space R_d , then prove that A contains only a finite number of points.

4. Attempt any TWO : (10)

(1) Prove that a closed subset of a complete metric space is complete.

(2) Prove that R^2 is a complete metric space ; with respect to the metric τ for R^2 defined as : $\tau\ (P,Q) = max\ \{ |\, x_1 - x_2 \,| \,, |\, y_1 - y_2 \,| \}$;
where $P = < x_1, y_1 > \ \&\ Q = < x_2\,, y_2 >$ in R^2 .

(3) Define a contraction mapping. Prove that a mapping

$$T : \left\langle \left(0,\ \frac{1}{3}],|\cdot|\right) \right\rangle \rightarrow \left\langle \left(0,\ \frac{1}{3}],|\cdot|\right)\right\rangle$$

defined by $Tx = x^2$; for every $x \in (0, \frac{1}{3}]$; is contraction, but it does not have a fixed point.

5. Attempt any TWO : (10)

(1) Define a compact metric space. Prove that a closed subset of a compact metric space is compact.

(2) Prove that: (i) Every finite set in any metric space is compact.
(ii) A connected subset of the metric space R_d is compact.

(3) If the metric space M has the Heine -Borel Property, then prove that M is compact.