RAN-0842

RAN-0842

B.Sc. (Sem-III) Examination

March / April - 2019

Mathematics Paper : MCS - 302

Discrete Mathematics - I

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **B.Sc. (Sem-III)**

Name of the Subject :

☛ **Mathematics Paper : MCS - 302**

Subject Code No.: **0 8 4 2**

Seat No.:

Student's Signature

(2) All questions are compulsory.
(3) Figures to the right indicate marks of corresponding questions.
(4) Follow usual notations.
(5) Use of non-programmable scientific calculator is allowed.

Q-1] Answer the following questions: **(10)**

1] For each pair of integers a and b, find integers q and r such that a = b q + r and $0 \leq r < b$.

(i) a = 221, b = 17 (ii) a = –1, b = 3

2] How many bytes are required to encode n bits of data where

(i) n = 500 (ii) n = 1001?

3] Find the generating function of the numeric function $a_k = 2$

4] Define algebraic structure and binary operation.

5] Define even permutation. Show that the permutation (6 5 4 3 1 2) is even.

Q-2] Answer any two of the following: (10)

1] Prove that if a and b are any two integers where $b>0$ then there exists unique integers q and r such that $a = b\,q + r;\ 0 \leq r < b$

2] Let gcd (a, b) = 1, then prove that gcd (a + b, a–b) = 1 or 2

3] Use prime factorization to find the gcd and *l*cm of

(i) 150, 70 (ii) 580, 8316

4] Find integers x and y such that $93x - 81y = 3$

Q-3] Answer any two of the following: (10)

1] Prove that (i) for all real numbers x and all integers m, $\lfloor x+m \rfloor = \lfloor x \rfloor + m$ and

(ii) for any real number *x*, if *x* is not an integer, then $\lfloor x \rfloor + \lfloor -x \rfloor = -1$

2] Define composition of functions. Let f and g be two functions from R to R where $f(x) = (3x-1)/5$ and $g(x) = (x^2+1)/2$. Find fog and gof.

3] If $f : R \to R$ is defined by $f(x) = 3x - 4 : x > 0$ and $f(x) = -3x + 2,\ x \leq 0$ then find

(i) f (0), f (2/3), f (–2) (ii) $f^1(0)$, $f^1(2)$, $f^1(-7)$.

4] Let x and y be two integers and suppose g (x, y) be recursively defined by

$$g(x,y) = \begin{cases} 5 & if\ x < y \\ g(x-y, y+2) + x & if\ x \geq y \end{cases}$$

then find g (2, 7), g (5, 3) and g(15, 2)

Q-4] Answer any two of the following: (10)

1] Find generating function for the sequence $\{a_k\}$ where

(i) $a_n = 3$ (ii) $a_n = 3^n$ (iii) $a_n = n + 3$

2] Find the generating function of the following sequences:

(i) 0, 1, – 2, 4, – 8, ... (ii) 0, 0, 1, 1, 1, 1, 1, ...

3] Use generating function to solve

$a_n - 9\,a_{n-1} + 20\,a_{n-2} = 0,\ a_0 = -3,\ a_1 = -10$

4] Write short notes on Tower of Hanoi.

Q-5] Answer any two of the following: (10)

1] Prove that the intersection of any two subgroups of a group (G, *) is again a subgroup of (G, *). Can we say that the union of two subgroups is also always a subgroup. Explain with illustration.

2] Show that {1, 2, 3} under multiplication modulo 4 is not a group but that {1, 2, 3, 4} under multiplication modulo 5 is a group.

3] Define a cyclic group. Show that the set $\{1, \omega, \omega^2\}$ is a cyclic group of order 3 with generators ω and ω^2 with respect of multiplication, ω being the cube root of unity.

4] Express the following as a product of transpositions and hence determine whether they are odd or even:

(i) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$
