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# RAN-0841

## S.Y.B.Sc. Sem-III Examination

## March / April - 2019

## Mathematics Paper: MCS-301

## Advanced Calculus - II

**Time: 2 Hours ]** **[ Total Marks: 50**

**સૂચના : / Instructions**

**નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.**
**Fill up strictly the details of ☛ signs on your answer book**

Name of the Examination:

☛ **S.Y.B.Sc. Sem-III**

Name of the Subject :

☛ **Mathematics Paper: MCS-301**

Subject Code No.: **0 8 4 1**

Seat No.:

Student's Signature

1) All questions are compulsory.
2) Figures to the right indicate marks of corresponding question.
3) Follow usual notations.
4) Use of non-programmable scientific calculator is allowed.

**Q-1. Answer the following questions:** **(10)**

1) Prove that $\Gamma(1/2) = \sqrt{\pi}$
2) Find $U_y$ and $U_z$ for $U(x,y,z) = \sqrt{x^2 + y^2 + z^2}$
3) If $f(x,y) = \dfrac{x^{1/2} + y^{1/2}}{x^{1/2} - y^{1/2}}$ then check whether f is homogeneous or not. Also find its degree.
4) Evaluate : $\int_0^\infty x^6 e^{-x} dx$
5) Find the sequence of partial sum of $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ....$

**Q2. Answer any two of the following: (10)**

1) Test the convergence of $\sum_{n=1}^{\infty} \frac{x^n}{n}$

2) For $n \in I^+$ let $S_n = \frac{1 \cdot 3 \cdot 5 \cdot \ldots (2n-1)}{2 \cdot 4 \cdot 6 \cdot \ldots (2n)}$, prove $\{S_n\}_{n=1}^{\infty}$ is non - increasing sequence.

3) Prove that $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ diverges.

4) Examine the convergence of $\sum_{n=1}^{\infty} (\sqrt{n} - \sqrt{n+1})$

**Q3. Answer any two of the following: (10)**

1) If H = f(y – z, z – x, x – y) then prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$

2) Find the limit $\lim_{(xy)\to(0,0)} \frac{xy}{x^2+y^2}$ if exists.

3) If z = f (u, v) and u = Ix + my, v = Iy – mx where I, m are constants then prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x^2} = (I^2 + m^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$

4) Obtain the expansion of $e^{ax}$ sin by in the form of powers of x and y.

**Q4. Answer any two of the following: (10)**

1) Find extreme values for $f(x, y) = x^3 + y^3 - 3x - 12y + 5$

2) Evaluate: $\int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{\pi} \sin(x + y)\, dx\, dy$

3) Show that $\sin(ax + by) = (ax + by) - \frac{(ax+by)^3}{3!} + \frac{(ax+by)^5}{5!} - \ldots$

4) Evaluate: $\int_0^4 \int_0^{\sqrt{16-x^2}} xy\, dx\, dy$

**Q5. Answer any two of the following: (10)**

1) Show that $\beta(I, m) = \frac{\lceil(I)\lceil(m)}{\lceil(I+m)}$

2) Evaluate : $\int_0^2 \frac{x^2}{\sqrt{2-x}} dx$

3) Show that $\frac{\beta(m, n+1)}{n} = \frac{\beta(m+1, n)}{m}$

4) Prove that $\int_0^1 \frac{x}{\sqrt{1-x^5}} dx = \frac{1}{5}\beta\left(\frac{2}{5}, \frac{1}{2}\right)$