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RAN-0950

S.Y.B.Sc. (Mathematics) (Sem IV) Examination

March / April - 2019

Paper-401 Advanced Calculus-II (Mathematics) (New)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **S.Y.B.Sc. (Mathematics) (Sem IV)**

Name of the Subject :

☛ **Paper-401 Advanced Calculus-II (Mathematics) (New)**

Subject Code No.: **0 9 5 0**

Seat No.:

Student's Signature

(1) Figures to the right indicate marks of the question.

(2) Follow usual notations and conventions.

Q. 1 **Answer any Five from the following.** **[10]**

1. Define local minimum value for a bivariate function.
2. Find f_{xx} for the function $f(x, y) = \sin x \sin y \sin (x + y)$ where $0 \leq x, y \leq \frac{x}{2}$.
3. Evaluate : $\int_0^2 \int_0^{\frac{y}{2}} y \, dydx$
4. Evaluate : $\int_0^x \int_0^{\sin x} y \, dxdy$
5. Evaluate : $\int_0^\infty x^4 e^{-x} dx$
6. Define Γn and evaluate $\Gamma 1$.

7. Find $L^{-1}\left[\frac{3p}{p^2+16}\right]$.

8. Find $L[t^7]$.

Q. 2 **Attempt any Two.** **[10]**

(a) Find three positive numbers whose sum is 30 and the product is maximum.

(b) Discuss about the extreme points of the bivariate function $f(x, y) = x^3 y^2 (1 - x - y)$.

(c) Find the point P(x, y) from which the sum of the distances from the axes and the line $x + y = 8$ is minimum.

Q. 3 **Attempt any Two.** **[10]**

(a) In usual notations prove that $B(l, m) = \frac{\Gamma l \, \Gamma m}{\Gamma(l+m)}$.

(b) Define the Beta integral and prove that $B(l,m) = \int_0^\infty \frac{y^{(m-1)}}{(1+y)^{(l+m)}}\, dy$.

(c) Prove that :

(i) $\int_0^\infty x^6 e^{-2x}\, dx = \frac{45}{8}$

(ii) $\int_0^1 \frac{x\, dx}{\sqrt{1-x^5}} = \frac{1}{5} B\left(\frac{2}{5}, \frac{1}{2}\right)$

Q. 4 **Attempt any Two.** **[10]**

(a) Change the order of the double integral $\int_0^2 \int_{\frac{x^2}{4}}^{3-x} f(x,y)\, dxdy$.

(b) Find area enclosed between the curves $y^2 = 2x$ and $x^2 = 2y$ using double integral.

(c) Evaluae $\int_0^1 \int_0^{\sqrt{1-x^2}} x^2 y^2\, dxdy$

Q. 5 **Attempt any Two.** **[10]**

(a) If $L^{-1}[\, f(p)] = F(t)$ then prove that $L^{-1}[f(mp - n)] = \frac{1}{m} e^{\frac{nt}{m}} F\left(\frac{t}{m}\right); m > 0$.

(b) Prove that $L[F(t)] = \frac{1 + (p-1)\, e^{-4p}}{p^2}$; where $F(t) = \begin{cases} l & , 0 < l < 4 \\ 5 & , t > 4 \end{cases}$

(c) Evaluate :

(i) $L[2t^3 + 5e^{-t}]$

(ii) $L^{-1}\left[\frac{1}{(p+1)(p-2)}\right]$