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RAN-7033

S.Y.B.Sc. (Semester-4) Examination

March / April - 2019

MTH-401 Advanced Calculus-II (Mathematics) (OLD)

(Old or New to be mentioned where necessary)

[Total Marks: 50

સૂચના : / Instructions

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **S.Y.B.Sc. (Semester-4)**

Name of the Subject :

☛ **MTH-401 Advanced Calculus-II (Mathematics) (OLD)**

Subject Code No.: **7 0 3 3**

Seat No.:

Student's Signature

Instruction:

(1) All questions are compulsory.

(2) Figures to the right indicate marks of the corresponding question.

Q:1 Answer the following : (Any five) **(10)**

1. Define Inverse Laplace Transform.
2. Show that B (m, n) = B (m+1, n) + B (m, n+1)
3. Find Laplace transform of the function F(t) = 1.
4. State the sufficient conditions for an extreme point of a bivariate function.
5. Evaluate $\int_0^1 x^2 (1-x)^3 dx$.
6. Evaluate $L^{-1}\left[\frac{2p+1}{p(p+1)}\right]$.
7. State the change of scale property of Inverse Laplace Transform.
8. Define Beta and Gamma integral.

Q:2 **Answer the following : (Any five)** **(10)**

1. Show that $f(x, y) = 2(x^2 - y^2) - x^4 + y^4$ attains minimum value at point $(0, -1)$. Also find its minimum value.
2. Discuss about the extreme values of $f(x, y) = x^2 y^2 - 5x^2 - 8xy - 5y^2$ at (0,0).
3. Find three positive real numbers whose sum is 15 and product is maximum.

Q:3 **Answer the following : (Any two)** **(10)**

(a) Prove the following : (i) $\sqrt{n} = 2\int_0^\infty e^{-y^2} y^{2n-1} dy$. (ii) $\sqrt{n} = \frac{1}{n}\int_0^\infty e^{-y^{\frac{1}{n}}} dy$.

(b) Evaluate $\int_0^2 x(8 - x^3)^{\frac{1}{3}} dx$

(c) Prove the following : (i) $\int_0^1 x^{m-1}(1 - x^2)^{n-1} dx = \frac{1}{2}B\left(\frac{m}{2}, n\right)$

(ii) $B(l, m) = B(m, l)$.

Q:4 **Answer the following : (Any two)** **(10)**

(a) State and prove Linearity property of Laplace Transform.

(b) Evaluate $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t} dt$

(c) Find Laplace tranformation of

$$F(t) = \begin{cases} \cos\left(t - \frac{2}{3}\pi\right), & t > \frac{2}{3}\pi \\ 0, & t < \frac{2}{3}\pi \end{cases}.$$

Q:5 **Answer the following : (Any two)** **(10)**

(a) In usual notation prove that $L^{-1}[f(p)] = F(t) \Rightarrow L^{-1}[f(p\text{-}a)] = e^{at} L^{-1}[f(p)]$.

(b) Evaluate : (*i*) $L^{-1}\left[\frac{3p + 7}{p^2 - 2p - 3}\right]$ (*ii*) $L^{-1}\left[\frac{1}{(p + a)^n}\right]$, $n \in N$

(c) Find $L^{-1}\left\{\frac{2 - 4p}{9p^2 + 16} + \frac{8 + 4p}{16p^2 - 9} + \frac{5}{2p - 3}\right\}$

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