RAN-0952

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S.Y.B. Sc. (Sem.-IV) Examination

March / April - 2019

Mathematics Paper : MTH - 403

Introduction To Abstract Algebra

(New Course)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **S.Y.B. Sc. (Sem.-IV)**

Name of the Subject :

☛ **Mathematics Paper : MTH - 403**

Subject Code No.: 0 9 5 2

Seat No.:

Student's Signature

Instructions :

(1) All questions are compulsory.
(2) Figures to the right indicate marks of corresponding question.
(3) Follow usual notations.
(4) Use of non-programmable scientific calculator is allowed.

1. Answer the following as directed : (Any FIVE) **(10)**

(1) If $a|b$ and $a|c$, then prove that $a|b.\, x + c.\, y$, for any integers x, y.
(2) If $a.\, b = a.\, c\ (mod\ n)$ and $(a, n) = 1$, then prove that $b \equiv c\ (mod\ n)$.
(3) In a group G; prove that $(a^{-1})^{-1} = a$; for every $a \in G$,
(4) If $x = x^{-1}$; for every element x in a group G, then prove that G is abelian.
(5) Justify : The set of all prime numbers is a subgroup of the group all positive rational numbers under multiplication.
(6) Prove that a cyclic group is abelian.
(7) Define a field. Give an example of an integral domain which is not a field.
(8) In a Boolean ring R; prove that $a + a = 0$; for every a in R.

2. **Attempt any TWO :** **(10)**

(1) Define a prime number. If a|$b.c$ and a and b are relatively prime integers, then prove that $a|c$.

(2) If $a \equiv b$ ($mod\ n$) and $c \equiv d$ ($mod\ n$), then prove that
(i) $a - c \equiv b - d$ ($mod\ n$); (ii) $a.c \equiv b.d$ ($mod\ n$).

(3) Find the integers m and n satisfying
(6540, 1206) = 6540 m + 1206 n.

3. **Attempt any TWO :** **(10)**

(1) In a group G; prove that:

(i) Every element $a \in G$ has a unique inverse.

(ii) $(a \bullet b)^{-1} = b^{-1} \bullet a^{-1}$ *for all a, b* $\in G$.

(2) Prove that $G = \{1, 3, 5, 7\}$ is a group under the binary operation x8 ; multiplication modulo 8 .

(3) Let G be a group. Prove that G is abelian if and only if
$(a.\ b)^2 = a^2 . b^2$; *for all a, b* $\in G$.

4. **Attempt any TWO :** **(10)**

(1) If H is a finite non - empty subset of a group G ; which is closed under the product in G, then prove that H is a subgroup.

(2) If H is a subgroup of a group G and $a \in G$, then prove that
$aHa^{-1} = \{a.h.\ a^{-1} \in G \mid h \in H\}$ is a subgroup of G .

(3) Define the order of an element in a group. In a group G;
if $a^3 = e$ and $a.\ b.\ a^{-1} = b^2$; for some $a, b \in G$, then find o(b).

5. **Attempt any TWO :** **(10)**

(1) Define an integral domain. Prove that every field is an integral domain.

(2) Prove that the commutative ring D is an integral domain if and only if $a,b,c \in D$ with $a \neq 0$; the relation

$a.b = a.c \Rightarrow b = c$ holds in D.

(3) Prove that every Boolean ring is commutative.
