RAN-1184

Third Year B. Sc. (Mathematics) (Sem. VI) Examination

March / April - 2019

MTH-602-Linear Algebra-II

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **Third Year B. Sc. (Mathematics) (Sem. VI)**

Name of the Subject :

☛ **MTH-602-Linear Algebra-II**

Subject Code No.: 1 1 8 4

Seat No.:

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate marks of the questions.

(4) Follow usual notations.

Q. 1. Answer the following questions (Any Five). **(10)**

(1) Define : Identity map in a vector space .Prove that it is linear .

(2) Is a transformation $T: V_2 \to V_3$ defined by $T(1,1) = (1,0,0)$, $T(2,1) = (0,0,1)$ and $T(0,1) = (0,1,0)$ linear? Justify your answer.

(3) Obtain the general rule for a linear map

$T : V_2 \to V_4$; $T(1,1) = (1,1,0,0)$, $T(1, -1) = (0,0,0,0)$.

(4) Is the following linear transformation one-one ? Justify your answer.

$T : V_2 \to V_4$; $T(1,1) = (1,0,0,0)$, $T(1,2) = (2,0,0,0)$.

(5) Prove that a linear transformation $T : V_2 \to V_2$ define by $T(x,y) = (x, -y)$ is non singular.

(6) Prove that A linear transformation $T : U_p \to V_p$ with $r(T) = p$ then $n(T) = 0$.

(7) Find the range and rank of the matrix $\begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix}$.

(8) State two difference between Orthogonal and Orthonormal set in an Inner product space V.

Q. 2. Answer the following (Any two). (10)

(1) State and prove necessary and sufficient condition for a linear transformation $T:U \to V$ be 1-1.

(2) Define Null space. Let $T:U \to V$ be a linear map. If $u_1 u_2, u_3, \ldots \ldots, u_n$ are linearly independent vectors of U with $N(T) = \{\theta_U\}$ then prove that $T(u_1), T(u_2), T(u_3) \ldots \ldots T(u_n)$ are L.I.

(3) Obtain the general rule of the linear transformation $T : V_3 \to V_3$ defined by $T(0,l,2) = (3,1,2)$, $T(1,1,1) = (2,2,2)$, $T(0,1,3) = (3,1,2)$.

Q. 3. Answer the following (Any two). (10)

(1) Verify Rank -Nullity theorem for a linear transformation $T: V_3 \to V_3$ defined by

$T(e_1) = e_1 - e_2$, $T(e_2) = 2e_2 + e_3$ and $T(e_3) = e_1 + e_2 + e_3$.

(2) Let $T:U \to V$ and $S:V \to W$ be two linear maps. Then prove that

(a) If ST is one-one, then T is one-one.

(b) If ST is non singular, then T is one-one and S on-to.

(3) Find the inverse of the linear transformation $T: V_3 \to V_3$ defined by $T(e_1) = e_1 + e_2$, $T(e_2) = e_2 - e_2 + e_3$ and $T(e_3) = 3e_1 + 4e_3$.

Q. 4. Answer the following (Any two). (10)

(1) Find the matrix $(T; B_1B_2)$ associated with a linear transformation $T : V_3 \to V_2$ defined by $T(X,Y,Z) = (2X + Y, 2Y - Z)$ relative to basis $B_1 = \{(1,1,0), (1,0,1), (0,1,1)\}$ and $B_2 = \{(1,1), (1,-1)\}$.

(2) Verify Rank-Nullity theorem for the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \\ -1 & 1 & 0 \end{bmatrix}$

(3) Find the Linear transformation T associated with a matrix $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$ relative to basis $B_1 = \{(1,-1,1), (1,2,0), (0,-1,0)\}$ and $B_2 = \{(1,0), (2,-1)\}$.

Q. 5. Answer the following (Any two). (10)

(1) In an Inner product space V, prove that

(a) $\|u + v\| \le \|u\| + \|v\|$, $\forall\, u, v \in V$.

(b) $u \cdot (\alpha v) = \overline{\alpha}\,(u \cdot v)$, $\forall\, u, v \in V$ and α a scalar.

(2) (a) Prove that any orthogonal set of non zero vectors in an inner product space V is L.I.

(b) Explain : Euclidean space and Unitary space in inner product space.

(3) Orthonormalized the L.I set $\{(0,0,1),(1,1,0),(1,5,2)\}$ by Gram Schmidt's process.
