RAN-1185

RAN-1185

T.Y.B.Sc.(Mathematics) (Sem-VI) Examination

March / April - 2019

Paper-603 Real Analysis III

(Old or New to be mentioned where necessary)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book
Name of the Examination:
☛ **T.Y.B.Sc.(Mathematics) (Sem-VI)**
Name of the Subject :
☛ **Paper-603 REAL ANALYSIS III**
Subject Code No.: 1 1 8 5

Seat No.:

Student's Signature

(2) Figures to the right indicate marks of the question.

(3) Follow usual notations and conventions.

Q.1 Answer any FIVE from the following. **[10]**

1. Define convergence and conditional convergence of a series of real numbers.
2. Define (i) Harmonic series (ii) Alternating series.
3. Give statement of the "ROOT TEST" for the absolute convergence of the series of real numbers.
4. Prove that the series $\sum_{n=2}^{\infty} \frac{1}{n^2}$ is convergent.
5. Prove that a singleton set is of measure zero.
6. Define Upper Riemann Integral and Lower Riemann Integral for a bounded function f on $[a, b]$.

7. If $f(x) = \int_0^x \sqrt{t + t^6}\, dt \; (x > 0)$ then find $f'(2)$.

8. If f is continuous on $[a, b]$, then prove that there exists c $\in (a, b)$ such that $\int_a^b f(x)\,dx = f(c)(b - a)$.

Q.2 Attempt any Two. **[10]**

(a) If $\sum_{n=2}^{\infty} a_n$ is a convergent series then prove that $\lim_{n \to \infty} a_n = 0$.
Is converse true ? Justify your answer.

(b) Check the convergence of following series.

(i) $1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots$ (ii) $\sum_{n=1}^{\infty} \frac{(n+1)}{(n+2)}$

(b) Prove that the series $(1 - 2) - (1 - 2^{\frac{1}{2}}) + (1 + 2^{\frac{1}{3}}) - (1 - 2^{\frac{1}{4}}) + \ldots$ converges.

Q.3 Attempt any Two. **[10]**

(a) If $\{a_n\}_{n=1}^{\infty}$ is a non-increasing sequence of positive numbers and if $\sum_{n=1}^{\infty} 2^n a_{2n}$ di-verges then prove that $\sum_{n=1}^{\infty} a_n$ diverges.

(b) Using appropriate test of convergence check the convergence for the series $\sum_{n=1}^{\infty} \frac{3}{4 + 2n}$

(c) For what, values of x does the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^x}$ converge ?

Q.4 Attempt any Two. **[10]**

(a) If each of the subsets $E_1, E_2, E_3, \ldots.$ of R^1 is of measure zero, then prove that $\bigcup_{n=1}^{\infty} E_n$ is also of measure zero.

(b) Prove that the characteristic function of the set of rational numbers on $[a, b]; a < b$ is not Riemann Integrable.

(c) Let $f(x) = x \; (0 \le x \le 1)$ and $\sigma_n = \left\{0, \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \ldots, \frac{n}{n}\right\}$ be any subdivision of [0,1] then compute $\lim_{n \to \infty} U|f ; \sigma_n|$

Q.5 Attempt any Two. [10]

(a) If f is a continuous function on the closed bounded interval $[a, b]$, and if $\Phi'(x) = f(x)\ (a \le x \le b)$, then prove that $\int_a^b f(x)\,dx = \Phi(b) - \Phi(a)$.

(b) If f is a continuous on $[a, b]$ and if $F(x) = \int_a^x f(t)\,dt\ (a \le x \le b)$, then prove that F is also continuous on $[a, b]$.

(c) If $f \in R[a, b]$, then prove that $|f| \in R[a,b]$ and $\left|\int_a^b f\right| = \int_a^b |f|$.
