RAN-1042

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T.Y.B.Sc. (Mathematics) (Sem. V) Examination

March / April - 2019

Paper - 503 Real Analysis I

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:
☛ **T.Y.B.Sc. (Mathematics) (Sem. V)**

Name of the Subject :
☛ **Paper - 503 Real Analysis I**

Subject Code No.: **1 0 4 2**

Seat No.:

Student's Signature

(1) All questions are compulsory.
(2) Digits to the right of each question indicate its marks.
(3) Follow usual symbols.

Q. 1 Answer any FIVE from the following. **[10]**

1) Define the upper bound of a set and find g.l.b. for the set $\{\pi+1, \pi+2, \pi+3, \pi+4, ...\}$

2) Prove that if $\{|S_n|\}_{n=1}^{\infty}$ converges to 0 then $\{S_n\}_{n=1}^{\infty}$ converges to 0.

3) If $S=\{S_n\}_{n=1}^{\infty}=\{2n-1\}_{n=1}^{\infty}$ and $N=\{n_i\}_{i=1}^{\infty}=\{i^2\}_{i=1}^{\infty}$ then find S_8 and S_{n_4}.

4) If $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers, if $S_n \leq M\ (n \in I)$ and if $\lim_{n\to\infty} s_n = L$ then prove that $L \leq M$.

5) Classify the sequence $\left\{n \sin \frac{\pi}{n}\right\}_{n=1}^{\infty}$ into

(A) convergent (B) divergent to ∞

(C) divergent to $-\infty$ (D) oscillating.

6) Define limit superior for a sequence of real numbers and find it for 12,–3,1,2,–3,12,–3,1,2,–3,........

7) Define a Cauchy sequence of real numbers with an illustration.

8) Give an example of a sequence $\{S_n\}_{n=1}^{\infty}$ which is not bounded but for which $\lim_{n \to \infty} \frac{S_n}{n} = 0$.

Q. 2 Answer any TWO from the following. [10]

1) Show that the set of all ordered pairs of integers is countable.

2) Prove that the set $A_n = \left\{ \frac{m}{n} : m \in I \right\}$ is countable for each $n \in N$, where I is the set of integers and use it to show that the set of all rational numbers is countable.

3) Define finite set with an illustration and prove that if B is an infinite subset of a countable set A, then B is also countable.

Q. 3 Answer any TWO from the following. [10]

1) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L, then prove that my subsequence of $\{S_n\}_{n=1}^{\infty}$ is also convergent to L.

2) Suppose $\{S_n\}_{n=1}^{\infty}$ converges to L then prove that $\{(-1)^n S_n\}_{n=1}^{\infty}$ converges to 0 if L = 0 and $\{(-1)^n S_n\}_{n=1}^{\infty}$ oscillates if $L \neq 0$.

3) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ converges to L, then prove that $\{S_n\}_{n=1}^{\infty}$ cannot also coverge to a limit distinct from L.

Q. 4 Answer any TWO from the following. [10]

1) Define a bounded sequence of real numbers and if the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent, then prove that $\{S_n\}_{n=1}^{\infty}$ is bounded.

2) Define a non increasing sequence of real numbers.

   For $n \in I$ let $t_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots\ldots + \frac{1}{n!}$ then prove that $\{t_n\}_{n=1}^{\infty}$ is monotone.

3) For $n \in I$ let $S_n = \frac{1 \cdot 3 \cdot 5 \cdot \ldots\ldots \cdot (2n-1)}{2 \cdot 4 \cdot 6 \cdot \ldots\ldots \cdot 2n}$ then prove that $\{S_n\}_{n=1}^{\infty}$ is convergent.

Q. 5 Answer any TWO from the following. **[10]**

1) If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers that diverges to infinity then prove that $\{S_n + t_n\}_{n=1}^{\infty}$ and $\{S_n \cdot t_n\}_{n=1}^{\infty}$ also diverge to infinity.

2) Define convergent sequence of real numbers and prove that every convergent sequence is Cauchy sequence.

3) Let $\{S_n\}_{n=1}^{\infty}$ diverges to infinity and if $\{t_n\}_{n=1}^{\infty}$ converges then prove that $\{S_n + t_n\}_{n=1}^{\infty}$ diverges to infinity.
