RAN-1041

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Third Year B.Sc. (Mathematics) (Sem.V) Examination

March / April - 2019

Linear Algebra-1

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **Third Year B.Sc. (Mathematics) (Sem.V)**

Name of the Subject :

☛ **Linear Algebra-1**

Subject Code No.: 1 0 4 1

Seat No.:

Student's Signature

1. All Questions are compulsory.
2. Figures to the right indicate marks of the questions.
3. Follow usual notations.

1. Answer the following questions Any Five. **(10)**

1) Is a binary operation $a * b = a$ associative? Justify.
2) In a vector space V, prove that $(-1)u = -u$ for every $u \in V$.
3) Let $M = \{\alpha u_0 / u_0 \in V, \alpha \text{ be a scalar}\}$. Is M be a subspace of a vector space$V$? Justify.
4) Find the spanof$X$ - axis and the plane $x+y = 0$ in V_3.
5) Prove that any set of vectors containing a zero of the space is always L.D.
6) Determine a basis for [(1,2,3), (3,1,0), (-2,1,3)].
7) Is a basis can never include the zero vector? Justify your answer.
8) Extend the L.I set {(1,2)} to two different basis of V_2.

2. Answer the following (Any two). [10)

1 Prove that : The Set $\mathbb{R}_3$ of all three tuples of real numbers is a real vector space.

2. Determine which of the following set is subspace of V_3.

   (a) $\{(x_1, x_2, x_3) \in V_3 / x_1 = 2x_2 \ or \ x_3 = 3x_2\}$

   (b) $\{(x_1, x_2, x_3) \in V_3 / x_1 \cdot x_2 = 0\}$

3. Prove that the set $\{\alpha_1 u_1 + \alpha_2 u_2 + \alpha_3 u_3 + + + + \alpha_n u_n / \alpha_i \in R, u_i \in V, i = 1 \ to \ n\}$ be a subspace of a vector space V.

3. Answer the following (Any two). (10)

1) If U and W are two subspaces of a vector space V, then prove that $U + W$ is a subspace of V and $U +$ W $= [U \cup W]$.

2) Let U and W be two subspaces of vector space V and $Z = U + W$. Then prove that Z = U ⊕ W if and only if any vector $z \in Z$ can be expressed uniquely as the form $z = u + w, u \in U, w \in W$.

3) If U and W are two subspaces of a vector space V then prove that $U + W = U$ if and only if $W \subset U$.

4. Answer the following (Any two). (10)

1) In a vector space V, prove that

   a) If v *is* a trivial linear combination of v_1, v_2,,,,,v_n then the set $\{v, v_1, v_2,,,,,,, v_n\}$ is L.D.

   b) If *u, v* and ware linearly independent vectors then the vectors $u + v$, $v + w$, $u+w$ are also L.I vectors.

2) (a) If a set is L.I then prove that any subset of it is also L.I.

   (b) Show that θ is collinear with any non zero vector v.

3) Prove that : In a vector space V, suppose $\{v_1, v_2, ..., v_n\}$ is an ordered set of vectors with $v_1 \neq \theta$. The set is L.D if and only if one of the vectors v_2, v_3, ..., v_n say v_k, belongs to the span of v_1, v_2, ...v_{k-1}.

5. **Answer the following (Any two).** **(10)**

1) Let $\{u_1, u_2, u_3, ..., u_n\}$ be the set of n L.I vectors in an n^{th} dimensional vectors space V then prove that the set $\{u_1, u_2, u_3, ... u_n\}$ is basis of V.

2) In a vector space V, the set $B = \{v_1, v_2, ,,,,,,,,, v_n\}$ generates V. Prove that the expression $v = \alpha_1 v_2 + \alpha_2 v_2 + - - - - + \alpha_n v_n$ is unique for every $v \in V$ if and only if the set is B *is* L.I.

3) Define : Basis in a vector space V. Prove that the subset.

   B={(1,1,1,), (1,-1,1), (0,1,1)} form a basis for V_3.
