



RAN-1040

T.Y.B.Sc. Sem-V Examination

March / April - 2019

Group Theory

Time: 2 Hours]

[Total Marks: 50

सूचना : / Instructions

(1)

नीचे दृशविले निशानीवाणी विगतो उत्तरवली पर अवश्य लपववी.
Fill up strictly the details of signs on your answer book

Name of the Examination:

T.Y.B.Sc. Sem-V

Name of the Subject :

Group Theory

Subject Code No.: 1 0 4 0

Seat No.:

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Student's Signature

- (2) All questions are compulsory.
- (3) Figures to the right indicate marks of the corresponding questions.
- (4) Follow usual notations.
- (5) Use of non-programmable scientific calculator is allowed.

Que 1: Answer any FIVE of the following:

(10)

- 1) Find the remainder when 5^{36} divided by 37.
- 2) True or False: If $a|x$ and $b|x$ then $(a+b)|x$. Justify your answer.
- 3) If G is a group such that $(a \cdot b)^2 = a^2 \cdot b^2$ for all $a, b \in G$, then show that G is abelian.
- 4) In S_3 give an example of $x, y \in S_3$ such that $(xy)^2 \neq x^2 y^2$
- 5) If $a \in G$ then define normalizer $N(a)$.

If $G = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} / a^2 + b^2 \neq 0; a, b \in R \right\}$ is a group and $\begin{pmatrix} 2 & 3 \\ -3 & 2 \end{pmatrix} \in G$

then find $N\left(\begin{pmatrix} 2 & 3 \\ -3 & 2 \end{pmatrix}\right)$.

- 6) Let $o(G) = 31$. Is G cyclic? Justify your answer.
- 7) If $o(G) = 24$ and H is a subgroup of G such that $o(H) = 12$. Is H a normal subgroup of G ? Justify your answer.
- 8) Is every subgroup of $Z(G)$ a normal subgroup of G ? Justify your answer.

Que 2: Answer any TWO of the following: (10)

- 1) If a, b are integers, not both 0, then prove that (a, b) exists. Moreover, show that there exists integers m_0 and n_0 such that $(a, b) = m_0a + n_0b$.
- 2) Prove that the relation “congruence modulo n ” defines an equivalence relation on the set of integers.
- 3) To check that n is a prime number, prove that it is sufficient to show that it is not divisible by any prime number p , such that $p \leq \sqrt{n}$.

Que 3: Answer any TWO of the following: (10)

- 1) If H is any arbitrary non-empty subset of a group G then state and prove the necessary and sufficient condition for H to be a subgroup of G .
- 2) Let G be the set of all symbols $a_i; i = 0, 1, 2, \dots, 6$ where $a_i \cdot a_j = a_{i+j}$; if $i + j < 7$ and $a_i \cdot a_j = a_{i+j-7}$ if $i + j \geq 7$.
Prove that G is an abelian group under the given operation.
- 3) In a group G , if $a, b \in G$ are any elements and $(ab)^n = a^n b^n$ for three consecutive integers n , then prove that G is abelian.

Que 4: Answer any TWO of the following: (10)

- 1) State and prove Lagrange's Theorem.
- 2) Prove that every subgroup of cyclic group is cyclic.
- 3) Let H be a subgroup of a group G and $a \in G$. If $aHa^{-1} = \{aha^{-1} \mid h \in H\}$, then show that aHa^{-1} is a subgroup of G . Also find $o(aHa^{-1})$, if H is finite.

Que 5: Answer any TWO of the following: (10)

- 1) State and prove fundamental theorem of homomorphism.
- 2) Let G be the group of integers under addition. Let N be the subgroup of G which contains all the multiples of 5. Find all elements of quotient group G/N . Also find index of N in G .
- 3) Define: Normal subgroup. Prove that a subgroup N is a normal subgroup of a group G if and only if $gNg^{-1} = N$; for every $g \in G$.