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RAN-1045

T.Y.B.Sc. (Sem. V) Examination

March / April - 2019

Mathematics : Paper - 506

(Number Theory)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **T.Y.B.Sc. (Sem. V)**

Name of the Subject :

☛ **Mathematics : Paper - 506**

Subject Code No.: 1 0 4 5

Seat No.:

Student's Signature

(1) Digits to the right indicates marks of the question.

(2) Follow the usual notations.

Q-l Answer any five questions: (10)

(1) If a is an odd integer, Show that gcd $(3a, 3a + 2) = 1$.

(2) Show that any composite three digit number must have a prime factor less than or equal to 31 .

(3) Check whether 2093 is a prime or not? If not write its prime power factorization.

(4) Verify whether integers 4, 11, 12, 13, 22, 82, 91 form a CRS (mod 7)

(5) Find 1cm (306, 657).

(6) For any integer a prove that $a^4 = 0$ *or* $1 \pmod 5$.

(7) Is $N = (447836)_{NINE}$ is divisible by 10.

(8) Solve: $18x + 81y = 26$.

Q-2 Answer any two questions : (10)

(1) Given integers a and b , not both of them zero then, show that there exists integers x and y such that gcd $(a, b) = ax + by$.

(2) If gcd (a, b, c) = gcd (gcd$(a, b), c)$ = gcd$(a$, gcd$(b, c))$ = gcd(gcd$(a, c), b)$ then find integers x,y,z such that gcd (198,288,512) = $198x + 288y + 512z$.

(3) For positive integers a and b, *prove that* gcd (a, b) *lcm* $(a,b) = ab$

Q-3 Answer any two questions : (10)

(1) Show that the linear Diophantine equation $ax + by = c$ has a solution if and only if d|c where d = gcd(a,b) *If* $x = x_0$, $y = y_0$ is a particular solution, then find all other solutions.

(2) Determine the all positive solutions of the Diophantine equation $221x + 35y = 11$.

(3) Prove that $1 + \sqrt{2}$ is an irrational number.

Q-4 Answer any two questions : (10)

(1) If $a \equiv b \pmod{n}$ then prove that gcd (a, n) = gcd (b, n).

(2) If p_n is the n^{th} prime number, then prove that $p_n \leq 2^{2^{n-1}}$.

(3) Find the remainder when 8888^{8888} is divided by 9 .

Q-5 Answer any two questions : (10)

(1) Let $N = a_m 9^m + a_{m-1} 9^{m-1} + a_{m-2} 9^{m-2} + --- + a_1 9^1 + a_0$,

be the representation of the positive integer N, with $0 \leq a_k \leq 8$, and $T = a_0 + a_1 + a_2 + a_3 + --- + a_m$. Then Prove that $8|N$ if and only if 8|T.

(2) Prove that the integer $1835^{1910} + 1986^{2061}$ is divisible by 7.

(3) Working modulo 9 or 11 find the missing digit x in the calculation $2x99561 = [3(523 + x)]^2$.
