RAN-1043

RAN-1043

T.Y.B.Sc (Sem-V) Examination

March / April - 2019

Mathematics Paper : MTH-504 : Real Analysis - II

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book
Name of the Examination:
☛ **T.Y.B.Sc (Sem-V)**
Name of the Subject :
☛ **Mathematics Paper : MTH-504 : Real Analysis - II**
Subject Code No.: 1 0 4 3

Seat No.:

Student's Signature

(2) All questions are compulsory.

(3) Figures to right indicate full marks of the corresponding questions.

(4) Follow usual notations.

1. Answer the following as directed (Any FIVE) **(10)**

(1) If $|x-3|<\frac{1}{10}$, then prove that $|x^2-x-6|<0.51$.

(2) If the real - valued functions f and g are continuous at $a \in R^1$, then prove that $f - g$ is also continuous at $a \in R^1$.

(3) Define: Metric for a Set & Equivalent Metrics.

(4) Justify: If ρ and σ are metrics for a set M, then ρ – σ is also a metric for M.

(5) Justify: The sequence $\left\{x_n=\frac{1}{n}\right\}_{n=1}^{\infty}$ is not convergent sequence of points in the metric space R_d .

(6) Construct the open balls B[2018 ; 0.2018] and B[0.2018; 2018] in the metric space R_d .

(7) Give an illustration such that an infinite intersection of open sets in a metric space is open.

(8) Justify: $\left(0, \frac{1}{2}\right]$ is open in the metric space $<[0,1], |\,.\,|>$.

2. Answer any TWO: (10)

(1) Let f and g be the real - valued functions and $a, L, M \in R$. If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$, then prove that $\lim_{x \to a} [f(x) - g(x)] = L - M$.

(2) Define the continuity of a real-valued function at $a \in R^1$. If the real-valued functions f and g are continuous at $a \in R^1$, then prove that max $\{f, g\}$ is also continuous at $a \in R^1$.

(3) Prove that $<R, d>$ is a metric space, where the function $d : R \times R \to [0, \infty)$ is defined as : $d(x, x) = 0$; for every $x \in R$ and $d(x, y) = 1$; for $x \neq y$ in R.

3. Answer any TWO: (10)

(1) Define a Cauchy Sequence in a metric space. If $\{x_n\}_{n=1}^{\infty}$ is a convergent sequence in the metric space Rd, then prove that there exists $N \in I$ such that $x_N = x_{N+1} = x_{N+2} = ...$, Also, give an example of a Cauchy sequence in the metric space Rd.

(2) Prove that a sequence of points in a metric space $< M, \rho >$ can not converge to two distinct points of M.

(3) Prove that the metircs σ and τ for R^2 defined respectively as follows:

$\sigma(P, Q) = |x_1 - x_2| + |y_1 - y_2|; \tau(P, Q) = max\{|x_1 - x_2|, |y_1 - y_2|\};$

Where $P = <x_1, y_1> \& Q = <x_2, y_2>$ in R^2;

are equivalent metrics for R^2.

4. Answer any TWO: (10)

(1) Define an open ball about a point a in R^1. Prove that the real-valued function f is continuous at a in R^1 if and only if given $\in > 0$ there exists $\delta > 0$ such that

$f^{-1}\{B[f(a); \in]\} \supseteq B[a; \delta]$

(2) Let $f : <M_1 , \rho_1> \to <M_2 , \rho_2>$ and $g : <M_2 , \rho_2> \to <M_3 , \rho_3>$ be two functions. If f is continuous at $a \in M_1$ and g is continuous at $f(a) \in M_2$, then prove that gof is continuous at $a \in M_1$.

(3) Define an open ball in the metric space R_d. Prove that every function from R_d into any metric space is continuous on R_d.

5. Answer any TWO: (10)

(1) (i) Define an open set in a metric space.

(ii) Prove that : (a) Every singleton set is not open in R^1 ;

(b) Any singleton set is open in R_d.

(2) Prove that an arbitrary union of open sets in any metric space is open.

(3) Let G be an open subset of the metric space R_1, then prove that x_G ; the characteristic function of G ; is continuous at each point of G.
