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DDD-0215

M. A. / M. Sc. (Mathematics) (Part - I) Examination

March / April - 2016

405 : Graph Theory & Discrete Structure

Time : Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **M. A. / M. Sc. (Mathematics) (Part - I)**

Name of the Subject :

☛ **405 : Graph Theory & Discrete Structure**

☛ Subject Code No. : 0 2 1 5 ☛ Section No. (1, 2,.....) : NIL

Seat No. :

Student's Signature

(2) Attempt all questions.

(3) Figures to the right indicate marks.

(4) Follow the usual notations and conventions.

1 (a) Prove that a given connected graph G is an Euler graph iff all vertices of G are of even degree. **7**

(b) Prove that an Euler graph G is arbitrarily traceable from vertex v in G iff every circuit in G contains v. **7**

(c) Show that in a simple graph with n vertices the maximum degree of any vertex is n–1 and the maximum number of edges is $\frac{n(n-1)}{2}$. **6**

OR

1 (a) Prove that any connected graph with n vertices and $n-1$ edges is a tree. **7**

(b) Prove that in a complete graph with $n(\geq 3)$ vertices, there are $\frac{n-1}{2}$ edge disjoint Hamiltonian circuits where n is an odd number. **7**

(c) Discuss the Utilities Problem. **6**

2 (a) Prove that Kuratowski's first graph is non-planar. **7**

(b) Prove that a graph can be embedded in a surface of the sphere iff it can be embedded in a plane. **7**

(c) Prove that a tree with more than two vertices will always has at least two pendant vertices. **6**

OR

2 (a) What is the minimum possible height of an n-vertex binary tree ? **7**

(b) Discuss the observation about the adjacency matrix. **7**

(c) Prove that in a connected graph G, the complement of a cut-set in G does not contain a spanning tree and the complement of a spanning tree does not contain a cut-set. **6**

3 (a) Prove that the ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-sets. **7**

(b) Let f from $\langle X, 0\rangle$ onto $\langle Y, \oplus\rangle$ be a homomorphism then prove that E_f is a congruence relation on $\langle X, 0\rangle$ given by $x_1 E_f x_2 \Leftrightarrow f(x_1) = f(x_2)$ for any $x_1, x_2 \in X$. **7**

(c) Let X be a non-empty set then prove that $\langle X^X, 0\rangle$ is a monoid, where $(fog)(x) = f(g(x))$ for $f, g \in X^X$ and $x \in X$. **6**

OR

3 (a) Define cancatenation and prove that for any alphabet $V, \langle V^*, 0\rangle$ is a monoid, where 'o' represents cancatenation. **7**

(b) Let B and A be the circuit and incidence matrix respectively, whose columns are arranged using the same order of edges. Then every row of B is orthogonal to every row of A. **7**

(c) Prove that any subgraph g of a connected graph G is contained in some spanning tree of G iff g contains no circuits. **6**

4 (a) For the set of natural numbers N, prove that $\langle N, +\rangle$ is a semigroup. Is the set odd non-negative integers form a subsemi group for $\langle N, +\rangle$? Justify your answer. **7**

(b) Prove that $\langle Z_m, +_m\rangle$ and $\langle Z_m^*, \oplus_m\rangle$ are isomorphic. **7**

(c) Prove that for a semigroup homomorphism commutativity is preserved. **6**

OR

4 (a) Use the tabular representation and circuit diagram representation to represent $f = \Sigma(0,1,2,3,13,15)$. **7**

(b) Define partial ordering relation. Give an example with justification. **7**

(c) Prove that the rank of a well-formed polish notation is 1. **6**

5 (a) Obtain the minimal expression for the function $f = \Sigma(5,7,10,13,15)$ Using Karnaugh map and Quine Mc-clusky algorithm. **7**

(b) Define minterm. Obtain the sum of product and product of sum canonical form for $a * b$ in the variables a, b and c. **7**

(c) Prove that the modular inequality holds in a lattice. **6**

OR

5 (a) Prove that the operations of meet and join on a lattice are commutative, associative, idempotent and satisfies the law of absorption. **7**

(b) Prove that the direct product of two distributive lattices a distributive lattice. **7**

(c) Show that, in a lattice 0 is the only complement of 1. **6**
