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DPP-0216

M. A. (Sem. II) (Mathematics) Examination

April/May - 2016

Paper : 406 : P.D.E. & Fourier Analysis

Time : Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **M. A. (SEM. II) (MATHEMATICS)**

Name of the Subject :
☛ **PAPER : 406 : P.D.E. & FOURIER ANALYSIS**

☛ Subject Code No. : **0 2 1 6** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) There are five questions in this question paper

(3) Answer all questions

(4) Figure to the right indicates marks of the questions

1 (a) Find the Orthogonal trajectories for the system of curves on a surface $f(x, y, z) = 0$. 7

(b) Explain any method to solve the set of equations $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ and Find the integral curves of the set of equations $\frac{dx}{x^2(y^3-z^3)} = \frac{dy}{y^2(z^3-x^3)} = \frac{dz}{z^2(x^3-y^3)}$ 7

(c) Find the equation of the integral surface of the differential equation $(x^2 - yz)p + (y^2 - zx)q = (z^2 - xy)$ which passes through the line x=1,y=0. 6

Or

1 (a) Show that the necessary and sufficient condition that the pfaffian differential equation $x \cdot dr = 0$ should be integrable is that $x. curl\ x = 0$. 7

(b) Show that the equation $xp - yq = x, x^2p + q = xz$ are compatible and find their solution. 7

(c) Find the complete integral of $x(1 + y)p = y(1 + x)q$. 6

2 (a) Derive the condition for two differential equations f(x,y,z,p,q)=0 and g(x,y,z,p,q)=0 to be compatible. 7

(b) Find the solution of $z^2 = pqxy$ by Jacobi's method. 7

(c) Find the partial differential equation corresponding to the sphere $x^2 + y^2 + (z - c)^2 = a^2$ having center $(0,0,c)$ with radius a. 6

Or

2 (a) If $(\alpha_r D + \beta_r D' + \gamma_r)^n$, $(\alpha_r \neq 0)$ is a factor of $F(D, D')$ and if the function $\emptyset_{r_1}, \emptyset_{r_2}, \ldots, \emptyset_{r_n}$ an arbitrary then $exp\left(-\frac{\gamma_r x}{\alpha_r}\right)\sum_{s=1}^{n} x^{s-1}\, \emptyset_{r_s}\left(\beta_{r_x} - \alpha_{r_y}\right)$ a solution of $F(D, D')z = 0$. 7

(b) Find the solution of the partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$. 7

(c) Find the equation of the system of surfaces which cut orthogonally the cones of the system $x^2 + y^2 + z^2 = cxy$. 6

Or

3 (a) State and prove charpit's method.

(b) Find the integral curves of the set of equation $(xz - y)dx + (yz - x)dy + (1 - z^2)dz = 0$.

(c) Determine the equation $yzdx + 2xzdy - 3xydz = 0$ is integrable and find the solution of which exists.

Or

3 (a) Derive the complex Fourier series for the interval $[c, c + 2l]$. 7

(b) Find the Fourier series for the function $f(x) = \left(\frac{\pi - x}{2}\right)^2$; $0 < x < 2\pi$. 7

(c) If the Fourier series of $f(x)$ over the interval $[-l, l]$, then derive the Parseval's identity for the given Fourier series in all possible cases. 6

4 (A) Derive the Fourier series of a periodic odd function $f(x)$ defined on $[-p, p]$ with period 2p. 7

(b) Find the complex form of Fourier series for the function $f(x) = e^{ax}, x \in (-\pi, \pi)$ in the form $e^{ax} = \frac{\sinh a\pi}{\pi} \sum_{n=-\infty}^{\infty} (-1)^n \frac{a+in}{a^2+n^2} e^{inx}$ 7

(c) Expand $f(x) = 1 + x, for\ 0 < x < 1$ as half range cosine series. 6

Or

4 (a) 1) Define Orthonormal system. Prove that every orthogonal system of functions is linearly independent system. 7

(b) Show that 7

(a) $\mathcal{F}_c\{e^{-ax}\} = \sqrt{\frac{2}{\pi}}\left(\frac{a}{a^2+k^2}\right)$; $a > 0$

(b) $\mathcal{F}_s\{e^{-ax}\} = \sqrt{\frac{2}{\pi}}\left(\frac{k}{a^2+k^2}\right)$; $a > 0$

(c) Find the complex form of Fourier series for the function $f(x) = \cos\ ax, x \in (-\pi, \pi)$ and deduce $\cos ax = \frac{a \sinh a\pi}{\pi} \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{a^2+n^2} e^{inx}$ 6

5 (a) Expand $f(x) = 2 - x, for\ 0 < x < 1$ as half range cosine series and sketch $f(x)$. 7

(b) Derive the Integral formula for partial sum of the fourier series. 7

(c) Prove that if $f(x)$ be an absolutely integrable function of period 2π then, at every discontinuity point where the right and left hand derivatives exists, the Fourier series of f converges to the value of $\frac{f(x+0)+f(x-0)}{2}$. 6

OR

5 (a) Find the Fourier transform of the function $f(x) = \begin{cases} 1 & ; |x| < 1 \\ 0 & ; |x| > 1 \end{cases}$ and hence evaluate the following 7

(a) $\int_0^\infty \frac{\cos xt \sin t}{t} \, dt$

(b) $\int_0^\infty \frac{\sin t}{t} \, dt$

(b) If function $f(x)$ satisfied Dinchlet's condition, then drive the fourier sine integral formula. 7

(c) Solve the Heat equation $u_t = u_{xx}$ subject to the following condition 6

(a) $u_x(0,t) = 0$

(b) $u(x,0) = \begin{cases} x & ; 0 \le x < 1 \\ 0 & ; x \ge 1 \end{cases}$

(c) $u(x,t)$ is bounded