



H-2325

First Year M. A. Examination

May/June - 2018

Mathematics : Paper-406

(P.D.E. & Fourier Analysis)

Time : 3 Hours]

[Total Marks : 100

Instructions :

(1)

नीचे दर्शावेक निशानीवाणी विगतो उत्तरवही पर अवश्य कर्तवी.
Fillup strictly the details of signs on your answer book.

Name of the Examination :
FIRST YEAR M. A.

Name of the Subject :
MATHEMATICS : PAPER-406

Subject Code No. : 2 3 2 5 Section No. (1, 2,.....) : Nil

Seat No. :

Student's Signature

- (2) There are five questions in the question paper.
- (3) Answer all questions.
- (4) Figure to the right indicates marks of the questions.

Q.1

- (a) Determine Simultaneous differential equations for the first order and the first degree in three variables. 7
- (b) Define: Pfaffian differential equation. Show that a Pfaffian differential equation in two variables always possesses an integrating factor. 7
- (c) Solve the equation $(x^2z - y^3)dx + 3xy^2dy + x^3dz = 0$ 6

OR

- (a) If X is a vector space such that $X \cdot \text{curl } X = 0$ and μ is an arbitrary function of x, y, z then prove that $\mu X \cdot \text{curl } \mu X = 0$ 7
- (b) show that the differential equation $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$ is integrable and also find its primitive. 7
- (c) State and prove Natani's method. 6

Q.2

- (a) Solve the equation $(y + z)dx + (z + x)dy + (x + y)dz = 0$ 7
- (b) Find the equation for integral surfaces passing through any given curve 7
- (c) Find the surface which intersects the surfaces of the system $z(x + y) = c(3z + 1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$. 6

OR

- (a) State and prove Charpit's method. 7
- (b) Find the surface which is orthogonal to the one-parameter system $z = cxy(x^2 + y^2)$ and which passes through the hyperbola $x^2 - y^2 = a^2, z = 0$. 7
- (c) If a characteristic strip contains at least one integral element of $F(x, y, z, p, q) = 0$ then show that it is an integral strip of the equation $F(x, y, z, z_x, z_y) = 0$. 6

Q.3

- (a) Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through the x-axis. 7
- (b) Find a complete integral of the equation $p^2y(1 + x^2) = qx^2$. 7
- (c) Describe Jacobi's method. 6

OR

- (a) If u_1, u_2, \dots, u_n are solution of the homogenous linear partial differential equation $F(D, D')z = 0$ then prove that $\sum_{r=1}^n c_r u_r$; where the c_r 's are arbitrary constants, is also a solution. 7
- (b) Find the particular integral of the equation $(D^2 - D')z = e^{2x+y}$. 7
- (c) Solve the equations $r + s - 2t = e^{x+y}$. 6

Q.4

- (a) Find the Fourier series for the function $f(x) = \left(\frac{\pi-x}{2}\right)^2; 0 < x < 2\pi$. 7
- (b) Expand the function $f(x) = e^x, -\pi < x < \pi$ in terms of Fourier series. 7
- (c) Derive the complex Fourier series for the interval $[0, 2\pi]$. 6

OR

- (a) Define orthonormal system of functions. Prove that the sum of the squares of the fourier coefficient of a square integrable function always converges. 7
- (b) Find the Fourier series to represent the function $f(x) = x^2, \text{ for } x \in [-\pi, \pi]$ and use Parseval's identity to show $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} \dots = \frac{\pi^4}{90}$. 7
- (c) Define odd and even functions. Discuss about half range sine and cosine fourier series. 6

Q.5

- (a) Prove the followings: 7
- 1) $\frac{1}{\pi} \int_{-\pi}^{\pi} D_n(u) du = 1$; where $D_n(u)$ is Dinchlet's kernel
 - 2) $D_n(u) = \frac{\sin(n+\frac{1}{2})u}{2 \sin(\frac{u}{2})}$
- (b) Define orthogonal system and complete orthonormal system of functions. Prove that if $\{\phi_n(x)\}$ be a complete orthonormal family of functions and f and g be integrable over $[a, b]$ then $f = g$ iff fourier expansion of f is equal to the fourier expansion of g . 7
- (c) Derive Fourier integral formula. 6

OR