



HJ-2387

M. A. (Mathematics) (Part - II) (External) Examination
May/June - 2018

501 - Differential Geometry & Linear Algebra

Time : 3 Hours]

[Total Marks : 100

Instructions :

(1)

नीचे दशांश देव निशानीवाणी विगतो उत्तरवही पर अवश्य लपवी. Fillup strictly the details of signs on your answer book. Name of the Examination : M. A. (Mathematics) (Part - II) (External) Name of the Subject : 501 - Differential Geometry & Linear Algebra Subject Code No. : 2 3 8 7 Section No. (1, 2,.....) : NIL	Seat No. : <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> Student's Signature
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- (2) All questions are compulsory.
- (3) Notations used are standard.
- (4) Each question is of 20 marks.

- 1 (a) Let M be a subspace of R^3 described by $x_1 - x_2 - x_3 = 0$. Find projection of $f=(2,3,2)$ on M by taking the standard inner product. 7
- (b) Given $f=(2, -2, -2,6)^T$ Find a vector S , obtained as a linear combination of vectors $v_1 = (1,1,1,1)^T$, $v_2 = (1, -1,0,0)^T$ and $v_3 = (1,1,0, -2)^T$ such that $\|f - s\|$ is minimal. 7
- (c) If $M=\text{span}\{A_1, A_2\}$ where $A_1 = (1,1,1, -1)^T$ and $A_2 = (-1,2,2,1)^T$ then find basis of M^\perp . 6

OR

- 1 (a) Define: Null space and Range of a linear operator. Prove that the null space and range of a linear operator are subspaces of norm space v and w respectively. 7
- (b) Let $A = [A_1, A_2, A_3, \dots, A_n]$. Let \langle , \rangle be the standard inner product on F^m then the following statement are equivalent 7
- I) $S = Ax$ is projection of y on M .
 - II) $Z=x$ minimizes $\|y - Az\|$ where $z \in F^n$.
 - III) X satisfies the normal equations $A^H x = A^H y$.

- (c) Define: Norm and Inner product space. Let $T: V \rightarrow W$ be a linear operator and $\dim(V)=n$ and $\dim(W) = m$. If T is invertible then prove that $m=n$. 6
- 2 (a) Let $T(x) = x'' + 2x' + x$, Find the matrix of T with respect to basis $p = \{1, t, t^2\}$. Find $N(T)$, $R(T)$ and solve $T(x) = 1 + t + t^2$. 7
- (b) Let $T: p_2 \rightarrow R^3$ be defined by $T(f) = X(X_0, X_1, X_2)^T$ where $x_k = \int_0^{k+1} f(t) dt; k=0, 1, 2$ then find $f(t)$. 7
- (c) (a) Let A be an $m \times n$ matrix and if \langle , \rangle is the standard inner product on F^m . Then prove that 6
- (1) $N(A) = R(A^H)^\perp$
 - (2) $R(A) = N(A^H)^\perp$
 - (3) $r(A) = r(A^H)$

OR

- 2 (a) Let $T: v \rightarrow v$ be a linear operator, $\beta = \{e_k\}$ and $\gamma = \{f_k\}$ where $k=1 \dots n$ be two bases for v . Let the change of basis from β to γ result in some matrix $p = [p_{ij}]$ then prove that $[T]_\gamma = p^{-1}[T]_\beta p$. 7
- (b) If T be a matrix operator $T(X) = AX$ with $A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 3 & -1 \\ -2 & 2 & 6 \end{bmatrix}$. 7
- Find a basis for $N(T)$ and its dimension. Also find $\text{rank}(T)$.
Is T one to one? Onto? Justify.
- (c) If $\{e_1, e_2, \dots, e_n\}$ is an orthogonal set and x, y are in span $\{e_1, e_2, \dots, e_n\}$ so that $x = \sum_{i=1}^n x_i e_i$ and $y = \sum_{i=1}^n y_i e_i$ for some x_i and y_i . Then prove that 6
1. The coefficients x_i is uniquely given by $x_i = \frac{\langle x, e_i \rangle}{\|e_i\|^2}$
 2. $\langle x, y \rangle = \sum_{i=1}^n x_i \bar{y}_i \|e_i\|^2$
- 3 (a) Define: Torsion and Screw-Curvature. A necessary and sufficient condition that a given curve is plane curve is that $\tau = 0$ at all points. 7