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JB-3117

Second Year B. Sc. (Sem. III) (CBCS) Examination

March/April – 2013

Discrete Mathematics : CCM-302 (CS)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fillup strictly the details of ☛ signs on your answer book.	Seat No. :
Name of the Examination : ☛ S. Y. B.Sc. (Sem. III) (CBCS)	
Name of the Subject : ☛ Discrete Mathematics CCM-302 (CS)	
☛ Subject Code No. : 3 1 1 7 ☛ Section No. (1, 2,.....) : Nil	Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate full marks.

1 Answer the following questions : **10**

(i) Define conditional proposition with illustration.

(ii) Define Tautology and Contradiction.

(iii) Prepare truth table for $(p \Rightarrow q) \wedge (q \Rightarrow p)$.

(iv) When does the inverse of any function exist ?

(v) Define floor and ceilling function with illustration.

(vi) Define normal group.

(vii) Define abelian group.

(viii) Define recurrence relation.

(ix) Explain homomorphism of group.

(x) Find characteristic equation of

$$a_r + 6a_{r-1} + 12a_{r-2} + 8a_{r-3} = 0.$$

2 (a) Use truth table to prove the associative law. **5**

$(p \vee a) \vee r = p \vee (q \vee r)$.

OR

(a) Let p : Meghal passes calculus.

q : Meghal is scholar

r : Meghal has the job.

Write out the following propositions (i) $p \Rightarrow q$,

(ii) $q \Rightarrow (p \wedge r)$, (iii) $r \vee (p \Rightarrow q)$, (iv) $\sim(p \Rightarrow q)$.

(b) Attempt any **two** : **10**

(i) State and prve DeMorgan's law.

(ii) State and prove Distributive law.

(iii) Find the principle disjunctive normal form without using the truth table. If (a) $p \vee q$, (b) $p \Leftrightarrow q$, (c) $\sim(p \wedge q)$, (d) $(p \wedge \sim r \wedge \sim s) \vee (r \wedge s)$.

(iv) Show that s is the valid conclusion from the give promises.

$p \Rightarrow \sim q$, $p \vee r$, $\sim s \Rightarrow p$, $\sim r$.

3 (a) Write a short note on Disarrangements. **5**

OR

(a) Write a short note on Fibonacci Sequence.

(b) Attempt any **two** : **10**

(i) Solve the difference equation $a_r = a_{r-1} + 6a_{r-2}$.

(ii) Suppoe * is a binary operation on a set A and e is the right identity element and f be the left identity element then show that $e=f$.

(iii) Find solution of the recurrence relation.

$a_r = 3a_{r-1} + 2r$.

(iv) Solve the difference equation $a_r = 8a_{r-1} + 10^{r-1}$.

4 (a) Let $f:R^+ \to R^+ \ : f(x)=\sqrt{x}$ and $g:R^+ \to R^+ \ : \ g(x)=3x+1$ **5**

then find *fog* and *gof*. Is *fog=gof* ?

OR

(a) Show that the following functions are primitive recursive functions : (i) $f(x)=x^2$, (ii) $f(n)=2^n$, (iii) $f(x,y)=|x-y|$.

(b) Attempt any **two** : **10**

(i) If $f:R\to R$ $f(x)=x^3-4x$ and $g:R\to R$ $g(x)=1/(x^2+1)$ and $h:R\to R$, $h(x)=x^4$ then find (a) $(fogoh)(x)$, (b) $(hogof)(x)$.

(ii) Find inverse of the following :

(a) $f:R\to R, f(x)=x^2+1$

(b) $f:R\to R, g(x)=x^2+3$

also find $\begin{matrix} f^{-1}(-8) & f^{-1}(17) \\ g^{-1}(7) & g^{-1}(19) \end{matrix}$.

(iii) Define composition of function

Find *fog* and *gof* for

$f:R\to R, f(x)=x^2+3$

$g:R\to R, g(x)=4x-5$

Is *fog* = *gof* ?

(iv) Suppose $f:A\to B$ and $g:B\to C$ are functions If both *f* and *g* are bijective then *gof* and *fog* are also bijective prove.

5 (a) Let H be a sub group of a then prove that $O(H)/O(G)$. **5**

OR

(a) Find the product of two permutations. Check whether it is commulative or not ?

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \quad g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}.$$

(b) Attempt any **two** : **10**

(i) Let G be an abelian group with identity element e then prove that $\forall\ x \in G$, satisfying the equation $x^2 = e$ form a sub group H of G.

(ii) Show that the operator * on the set $Q - \{1\}$ defined by $a * b = a + b - ab$ is closed, commutative and associative find identity and inverse elements.

(iii) Let G be a group and two relatively prime numbers $m \,\&\, n \in G$ such that $a^m \cdot b^m = b^m \cdot a^m$ and $a^n b^n = b^n \cdot a^n$ then prove that G is abelian.

(iv) Show that if $f : G \to G'$ is isomorphism then $f^{-1} : G' \to G$ is also an isomorphism.