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JB-3113

Second Year B. Sc. (Sem. III) (Computer Science) Examination

March/April – 2013

Advanced Calculus : CCM-301CS

Time : Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **S. Y. B.Sc. (Sem. III) (Computer Science)**

Name of the Subject :
☛ **Advanced Calculus : CCM-301CS**

☛ Subject Code No. : **3 1 1 3** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All the questions are compulsory.
(3) Digits shown on right hand side indicate full marks of the question.
(4) Attempt the questions as directed.

1 Do as directed : **10**

(i) Discuss the continuity of $f(x, y) = xy + 2$ at the point $(1, 2)$.

(ii) Evaluate $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$, where $u = \frac{x^3 - y^3}{x^2 y}$.

(iii) If $x^4 + y^4 - 4a^2 xy = 0$, then evaluate $\frac{dy}{dx}$.

(iv) Evaluate $\int_1^2 \int_0^1 xy^2 dydx$.

(v) Evaluate $\int_0^\infty e^{-x} x^5 dx$.

(vi) Evaluate $\lim_{(x,y)\to(1,3)} \left(x^2 + 2y\right)$.

(vii) Find the expansion of $\cos x \cos y$ in powers of x and y upto second order terms.

(viii) If $u = x^2 + y^2$, $x = s + 3t$, $y = 2s - t$ then find $\frac{\partial u}{\partial s}$.

(ix) Discuss the nature of the series $1-1+1-1+\ldots\infty$.

(x) State the sufficient condition for Maxima-Minima for the function of two variables.

2 (a) If $u=\sin\left(\sqrt{x}+\sqrt{y}\right)$, prove that **5**

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\left(\sqrt{x}+\sqrt{y}\right)\cos\left(\sqrt{x}+\sqrt{y}\right).$$

OR

(a) If $u=f(r)$, $r^2=x^2+y^2+z^2$, then prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=f''(r)+\frac{2}{r}\cdot f'(r)$, where $f'(r)$ denotes differentiation of f with respect to r.

(b) Attempt any **two**. **10**

(i) Expand $f(x,y)=\log(xy)$ in the powers of $(x-1)$ and $(y-1)$.

(ii) Find the point $P(x,y)$ from which the sum of squares of the distances from the axes and the line $x+y=8$ is minimum.

(iii) Discuss the continuity of $f(x,y)$ at the point $(0,0)$,

where $f(x,y)=\begin{cases}\frac{\tan x-\tan y}{\sin x-\sin y}, & \sin x\neq\sin y\\ 2, & \sin x=\sin y\end{cases}$.

(iv) If $u=\left(1-2xy+y^2\right)^{-1/2}$, prove that $x\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}=y^2u^3$.

3 (a) Evaluate $\iint_s xy\,dx\,dy$, where s is bounded by x-axis, $x=2a$ and $x^2=4ay$. **5**

OR

(a) Change the order of integration in $\int_0^1\int_{x^2}^{2-x} xy\,dx\,dy$ and hence evaluate the same.

(b) Attempt any **two.** **10**

(i) Show that $\int_0^2 x^4 \left(8-x^3\right)^{-\frac{1}{3}} dx = \frac{16}{3} B\left(\frac{5}{3}, \frac{2}{3}\right)$.

(ii) Prove that $\Gamma n = \int_0^1 \left[\log \frac{1}{y}\right]^{n-1} dy$.

(iii) Evaluate $\int_0^\infty e^{-h^2 x^2} dx$.

(iv) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

4 (a) Test the convergence of the series $\frac{1}{1\cdot 2\cdot 3} + \frac{3}{2\cdot 3\cdot 4} + \frac{5}{3\cdot 4\cdot 5} + \cdots$ **5**

OR

(a) Examine the convergence of $\frac{2}{1^p} + \frac{3}{2^p} + \frac{4}{3^p} + \cdots$.

(b) Attempt any **two** : **10**

(i) Test the convergence of the series whose n^{th} term is $\frac{n^2}{2^n}$.

(ii) Examine the convergence or divergence of the series whose n^{th} term is $\frac{1}{\sqrt{n} + \sqrt{n+1}}$.

(iii) Test the convergence of $\frac{1}{1+2} + \frac{2}{1+2^2} + \frac{3}{1+2^3} + \cdots$.

(iv) Test for convergence of the series $\sum \frac{2^n}{n^3}$.

5 (a) If $u = \sqrt{x^2 + y^2 + z^2}$, then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{2}{u}$. **5**

OR

(a) If $f(x) = x + 2$, defined in the interval $[0,5]$. Show that $f(x)$ is Riemann integrable on $[0,5]$ by dividing interval in 5 equal parts and hence evaluate $\int_0^5 (x+2)\,dx$.

(b) Attempt any **two.** **10**

(i) Show that the sequence whose n^{th} term $u_n = \frac{2n}{n+un^{1/2}}$ is convergent.

(ii) If C is constant function then using the definition of Riemann integration, show that function is Riemann integrable and $\int_a^b c\,dx = c(b-a)$.

(iii) Find the extreme values for $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$.

(iv) Evaluate $\iint_R (x^2 + y^2)\,dxdy$, where R is the region bounded by $x = 0$, $y = 0$ and $x+y=1$.
