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JB-3116

Second Year B. Sc. (Sem. III) (Mathematics) Examination

March/April – 2013

CCM-301 : Advanced Calculus - I

Time : Hours] [Total Marks :

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fillup strictly the details of ☛ signs on your answer book.	Seat No. :
Name of the Examination : ☛ S. Y. B. Sc. (Sem. III) (MATHEMATICS)	
Name of the Subject : ☛ CCM-301 Advanced Calculus - I	
☛ Subject Code No. : 3 1 1 6 ☛ Section No. (1, 2,.....) : Nil	Student's Signature

(2) All questions are compulsory.

(3) Figures to the **right** indicate marks of the corresponding question.

(4) Question 1 carries **10** marks and the remaining questions carry **15** marks each.

1 Answer the following : (any **five**) **10**

(i) Obtain $\lim_{x\to 1}\left\{\lim_{y\to 1}\frac{x^2+y^2}{x+y}\right\}$ and $\lim_{y\to 1}\left\{\lim_{x\to 1}\frac{x^2+y^2}{x+y}\right\}$.

(ii) U_x, U_y and U_z for $U(x,y,z)=\log\left(x^2+y^2+z^2\right)$.

(iii) If $x=\rho\cos\phi$, $y=\rho\sin\phi$, $z=z$ then find $\frac{\partial(x,y,z)}{\partial(\rho,\phi,z)}$

(iv) Define Jacobian of bivariate function.

(v) Prove that $f(x,y)=x^3+y^3$ is minimum at (0,0).

(vi) If $f_{xx}(a,b)=r$, $f_{xy}(a,b)=s$, $f_{yy}(a,b)=t$ then write down your conclusions regarding the different values of $rt-s^2$.

(vii) If $\vec{r}=(t)\hat{i}+(t+1)j+\left(t^2+t+1\right)k$ then find $\frac{d\vec{r}}{dt}$.

(viii) If $f=\frac{1}{2}\left(x^2+y^2+z^2\right)$ then find the value of grad f.

2 (a) If f is differentiable function in x and y such that $xf_x + yf_y = mf$ then prove that f is homogeneous of degree m. **5**

OR

(a) Let $\phi(x)$ be a function continuous at the point (a,b), where $b=\phi(a)$ and $\lim_{(x,y)\to(a,b)} f(x,y) = l$ then prove that $\lim_{x\to a} f(x,\phi(x)) = l$.

(b) Solve any **two** : **10**

(i) Discuss the continuity of the function $f(x,y)$ at the point (0,0) where

$$f(x,y) = \frac{\tan(x+y)}{x+y}, \quad x+y \neq 0$$
$$= 1, \quad x+y=0$$

(ii) If $z = f(x,y)$ and $x = e^{-u} + e^{v}$, $y = e^{u} + e^{-v}$ then prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = y\frac{\partial z}{\partial y} - x\frac{\partial z}{\partial x}$.

(iii) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u.$$

(iv) If $u = f(r)$, $r^2 = x^2+y^2+z^2$ then prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = f''(r) + \frac{2}{r}\cdot f'(r).$$

3 (a) Obtain expression of $e^{ax}\cos by$ in terms of x and y. **5**

OR

(a) Expand $f(x,y) = \log xy$ in the powers of $(x-1)$ and $(y-1)$.

(b) Solve any **two** : **10**

(i) If it is given that $y_1(x_1 - x_2) = 0$ and $y_2\left(x_1^2 + x_1x_2 + x_2^2\right) = 0$ then find $\frac{\partial(y_1, y_2)}{\partial(x_1, x_2)}$.

(ii) If $u^3 + v + w = x + y^2 + z^2$, $u + v^3 + w = x^2 + y + z^2$, $u + v + w^3 = x^2 + y^2 + z$ then prove that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = \frac{1 - 4(xy + yz + zx) + 16xyz}{2 - 3\left(u^2 + v^2 + w^2\right) + 27u^2v^2w^2}.$$

(iii) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, then find $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$.

(iv) Obtain expression of $\sin x \sin y$ in terms of x and y.

4 (a) Show that $f(x, y) = 2(x - y)^2 - x^4 - y^4$ has maximum value 8 at $\left(-\sqrt{2}, \sqrt{2}\right)$. **5**

OR

(a) Find extreme values of $u = x^3y^2(1 - x - y)$.

(b) Solve any **two** : **10**

(i) Find the interior point of the triangle, such that the sum of the squares of its distances from the vertices is minimum.

(ii) Show that $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ attains maximum value at point $(-1, -2)$. Also find its maximum value.

(iii) Discuss about the extreme points of $u = xy + a^3\left(\frac{1}{x} + \frac{1}{y}\right)$.

(iv) Find extreme values of $f(x, y) = x^3 + y^3 - 3x - 12y + 5$.

5 (a) Define gradient. In usual notations prove that $Curl\, grad\, f = \vec{0}$. **5**

OR

(a) If $\vec{V}$ is differentiable vector function and ϕ is differentiable scalar function, then prove that $div(\phi\vec{V}) = (grad\phi)\cdot\vec{V} + \phi(div\vec{V})$.

(b) Solve any **two** : **10**

(i) If $\vec{r} = (\sinh\lambda t)\vec{a} + (\cosh\lambda t)\vec{b}$ where $\vec{a}$ and $\vec{b}$ are constant vectors, then prove that $\frac{d^2\vec{r}}{dt^2} - \lambda^2\vec{r} = \vec{0}$.

(ii) If $\vec{r} = (t^2)\hat{i} - (t)j + (2t+1)k$ then find values of $\frac{d\vec{r}}{dt}$, $\frac{d^2\vec{r}}{dt^2}$, $\left|\frac{d\vec{r}}{dt}\right|$ and $\left|\frac{d^2\vec{r}}{dt^2}\right|$ at $t = 0$.

(iii) State the condition for a vector point function to be solenoidal. If $\vec{f} = (x^2y)\hat{i} + (xz)j + (2yz)k$ then find $Curl\,\vec{f}$. Check whether $Curl\,\vec{f}$. is solenoidal or not.

(iv) If $f = x^2y + 2xyz + z^2$, then prove that $Curl\,grad\,f = \vec{0}$.