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JB-3114

Second Year B. Sc. (Sem. III) Examination

March/April – 2013

Mathematics : CCM-302

(Differential Equations) (New Course)

Time : Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **Second Year B. Sc. (Sem. III)**

Name of the Subject :

☛ **Mathematics : CCM-302 (New Course)**

☛ Subject Code No. : **3 1 1 4** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All questions are compulsory.
(3) Digits to the right indicate marks of that question.
(4) Follow usual notations.

1 Attempt any **five** out of **eight** : **10**

(i) Obtain the complimentary function of

$$\frac{d^3y}{dx^3}-2\frac{d^2y}{dx^2}-\frac{dy}{dx}+2y=0.$$

(ii) Obtain the general solution of $\frac{d^2y}{dx^2}-y=\sin x$.

(iii) Convert $x^2\frac{dy}{dx}-2y=x$ into linear differential equation with constant coefficients.

(iv) Convert $(x+1)\frac{dy}{dx}+2y=(x+1)\cdot(x+2)$ into linear differential equation with constant coefficients.

(v) Write general solution of $\frac{dy}{dx}-P(x)y=Q(x)$.

(vi) Find $y(x)$ for $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=\sin x$.

(vii) Find the particular integral of

$$x\frac{d^2y}{dx^2}+6\frac{dy}{dx}+\left(x^2+1\right)y=x^2y.$$

(viii) Find the complimentary function of

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+(3+x)\,y=x\cdot y$$

2 (a) Describe the method of finding the particular integral of $f(D)\,y=\sin ax$, where $D\equiv\frac{d}{dx}$ and $f(D)=D^n+P_1D^{n-1}+\ldots+P_n$. $\left(P_1,P_2,\ldots,P_n\in R\right)$, $\phi\left(-a^2\right)\neq 0$. **5**

OR

(a) Describe the method of finding the particular integral of $f(D)\,y=e^{ax}v(x)$, where $D\equiv\frac{d}{dx}$ and $f(D)=D^n+P_1D^{n-1}+\ldots+P_n$ $\left(P_1,P_2,\ldots,P_n\in R\right)$, $f(D+a)\neq 0$. **5**

(b) Solve any **two** of the following **four** : **10**

(i) $\dfrac{d^2y}{dx^2}-y=2+5x$.

(ii) $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+3y=x^2+4x$.

(iii) $\dfrac{d^2y}{dx^2}+4y=\sin 3x+e^x+x^2$.

(iv) $\dfrac{d^2y}{dx^2}+3\dfrac{dy}{dx}+2y=e^{2x}\sin x$.

3 (a) Describe the method of finding the general solution of **5**

$$x^n\frac{d^ny}{dx^n}+P_1x^{n-1}\frac{d^{n-1}y}{dx^{n-1}}+P_2x^{n-2}\frac{d^{n-2}y}{dx^{n-2}}+\ldots+P_ny=X(x).$$

OR

(a) Describe the method of finding the general solution of **5**

$$(ax+b)^n \frac{d^n y}{dx^n} + P_1 (ax+b)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \ldots + P_n y = X(x).$$

(b) Solve any **two** of the following **four** : **10**

(i) $x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$

(ii) $x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 4y = x^2 \cdot \log x$

(iii) $x^2 \frac{d^2 y}{dx^2} - 5x \frac{dy}{dx} + 9y = x^5$

(iv) $(x+a)^2 \frac{d^2 y}{dx^2} - 4(x+a) \frac{dy}{dx} + 6y = x$.

4 (a) Describe the method of finding the general solution of **5**

$\frac{d^2 y}{dx^2} + P(x) \frac{dy}{dx} + Q(x) y = X(x)$ in terms of known integral.

OR

(a) Describe the method of finding the general solution of **5**

$\frac{d^2 y}{dx^2} + P(x) \frac{dy}{dx} + Q(x) y = X(x)$, when

$m(m-1) + Pmx + Qx^2 = 0 \Rightarrow m \in R$.

(b) Solve any **two** of the following **four** : **10**

(i) $x^2 \frac{d^2 y}{dx^2} - 2x(1+x) \frac{dy}{dx} + 2(1+x) y = 0$.

(ii) $\frac{d^2 y}{dx^2} + (1 - \cot x) \frac{dy}{dx} - y \cot x = \sin^2 x$

(iii) $x \frac{d^2 y}{dx^2} - (1+x) \frac{dy}{dx} + y = x^2 e^{2x}$

(iv) $\frac{d^2 y}{dx^2} - 2 \tan x \frac{dy}{dx} + 3y = 2 \sec x$ Where $y_1(x) = \sin x$.

5 (a) Explain the method of finding the general solution of **5**

$\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=X(x)$ by removal of first ordered derivative.

OR

(a) Explain the method of finding the general solution of **5**

$\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=X(x)$ by changing an independent variable.

(b) Solve any **two** of the following **four** : **10**

(i) $\frac{d^2y}{dx^2}+\frac{2}{x}\frac{dy}{dx}-16y=0$.

(ii) $\frac{d^2y}{dx^2}+\cot x\frac{dy}{dx}+4\operatorname{cosec}^2x\cdot y=0$.

(iii) $\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+\left(x^2+2\right)y=e^{\frac{1}{2}\left(x^2+2x\right)}$.

(iv) $\frac{d^2y}{dx^2}-2\tan x\frac{dy}{dx}+5y=0$.