# JB-3115

## Second Year B. Sc. (Sem. III) Examination

**March/April – 2013**

## Mathematics : Paper - CCM-303

*(Numerical Analysis - I)*

Time : 3 Hours] [Total Marks : **70**

**Instructions :**

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
**Fillup strictly the details of ☛ signs on your answer book.**

Name of the Examination :

☛ **S. Y. B. Sc. (Sem.-III)**

Name of the Subject :

☛ **Mathematics : Paper - CCM-303**

☛ Subject Code No. : **3 1 1 5** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) Figures to the right indicate marks of the question.

(3) Follow usual notations.

(4) Use of scientific non-gprogrammable calculator is allowed.

1 Answer any **five** as directed : **10**

(1) (i) Round-off the given numbers to two decimal places :

(a) 81.265

(b) 2.395.

(ii) Round-off the given numbers to four significant digits :

(a) 0.0022218

(b) 0.70025.

(2) Define :

(i) Absolute error,

(ii) Relative error.

(3) State the condition for 'the Iteration method' to succeed. Explain by one diagram of this method succeeds.

(4) Explain by two diagrams the failure of 'Newton-Raphson Method.'

(5) Find the value of $\nabla^3 y_3$.

(6) Prove that : $\Delta - \nabla \equiv \delta^2$.

(7) Prove that : $\mu\delta \equiv \frac{1}{2}\Delta E^{-1} + \frac{1}{2}\Delta$.

(8) Define : Interpolation.

2 (a) Prove that the magnitude of the absolute error of a sum of $n$ numbers does not exceed the sum of the absolute errors of the numbers. 5

**OR**

(a) Derive the formula for finding the absolute error in the product of two numbers. 5

(b) Attempt any **two** : 10

(1) Find the absolute error and the relative error of the number X=2.1, if both of the digits are correct.

(2) If an approximate value of $\pi$ is given by $X_1$=3.1428571 and its true value is X=3.1415926, then find the absolute error and the relative error.

(3) Estimate the absolute error in the product $a \cdot b$, where a=56.54 and b=12.4, which both are correct to significant digits given.

(4) Let $f(x) = x^3 - 3x^2 + 5x - 10$. Evaluate f(1) and find the Taylor series approximations of orders 0 and 1. Also, state the absolute error in each case.

3 (a) Explain 'the methods of False-Position' to obtain a real root of an equation $f(x)$=0. 5

**OR**

(a) Explain 'the 'Newton-Raphson Method' to obtain a real root of an equation f($x$)=0. **5**

(b) Obtain a real root of any two of the given equations; correct to four decimal places : **10**

(1) $x \cdot e^x - 1 = 0$ by 'the Bisection Method'.

(2) $x^3 + x^2 - 1 = 0$ by 'the Iteration method'.

(3) $x^3 - 2x + 5 = 0$ by 'the method of False-Position'.

(4) $\sin x = 1 - x$ by 'the Newton-Raphson method'.

**4** (a) Prove that : **5**

$$u_x - \Delta^n u_{x-n} = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + \dots\dots + \Delta^{n-2} u_{x-n+1} + \Delta^{n-1} u_{x-n} .$$

**OR**

(a) Find f(6), using the following table : **5**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y = f(x)$ | 8 | 12 | 19 | 29 | 42 |

(b) Attempt any **two** : 10

(1) Prove that :

(i) $y_n = y_o + nc_1 \Delta y_o + nc_2 \Delta^2 y_o + \dots\dots + \Delta^n y_o$

(ii) $\Delta^n y_o = y_n - nc_1 y_{n-1} + nc_2 y_{n-2} - \dots\dots\dots + (-1)^n y_o$

(2) Prove that :

(i) $\nabla \equiv 1 - E^{-1}$,

(ii) $\mu\delta \equiv \frac{1}{2}(\Delta + \nabla)$.

(3) Prove that :

(i) $\mu^2 \equiv 1 + \frac{\delta^2}{4}$;

(ii) $\Delta \log f(x) = \log\left(1 + \frac{\Delta f(x)}{f(x)}\right)$

(4) Find the first term of the sequence, whose second and subsequent terms are 8, 3, 0, –1, 0, ............ .

5 (a) Derive 'Newton's forward difference interpolation formula'. **5**

**OR**

(a) Derive 'Newton's Backward Difference Interpolation Formula. **5**

(b) Attempt any **two** : **10**

(1) Find $f(22)$, using the following table :

| $x$ | 20 | 25 | 30 | 35 | 40 |
|---|---|---|---|---|---|
| $y = f(x)$ | 23 | 26 | 30 | 35 | 42 |

(2) Find f(5), using the following table :

| $x$ | 0 | 2 | 4 | 6 |
|---|---|---|---|---|
| $y = f(x)$ | 1 | 1 | 33 | 145 |

(3) Using 'Gauss' forward formula, find $f(32)$ given that :

| $x$ | 25 | 30 | 35 | 40 |
|---|---|---|---|---|
| $y = f(x)$ | 0.2707 | 0.3027 | 0.3386 | 0.3794 |

(4) Using 'Gauss' backward formula, find $f(1946)$ given that :

| $x$ | 1931 | 1941 | 1951 | 1961 | 1971 |
|---|---|---|---|---|---|
| $y = f(x)$ | 15 | 20 | 27 | 35 | 52 |