JB-3193

Second Year B. Sc. (Sem. - IV) (Mathematics) Examination

April/May – 2013

CCM - 401 : Advanced Calculus - II

Time : 3 Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fillup strictly the details of ☛ signs on your answer book.	Seat No. :
Name of the Examination : ☛ S. Y. B. Sc. (Sem. - IV) (Mathematics)	
Name of the Subject : ☛ CCM - 401 Advanced Calculus - II	
☛ Subject Code No. : 3 1 9 3 ☛ Section No. (1, 2,.....) : Nil	Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate marks of the corresponding question.

(4) Question 1 carries 10 marks and the remaining questions carry 15 marks each.

1 Do as directed : (any five) **10**

(1) Check the validity of $\int_1^2\int_2^3 dx\,dy = \int_1^2\int_2^3 dy\,dx$.

(2) Find the value of $\int_0^1\int_0^y 2\,dy\,dx$.

(3) Evaluate $\int_0^\infty x^4 e^{-x} dx$.

(4) Find the value of $\int_0^1 x^4 (1-x)^3 dx$.

(5) Evaluate $L[1]$.

(6) Find the value of $L^{-1}\left[\frac{1}{(p+a)^n}\right] for\, n \in N$.

(7) Evaluate $L^{-1}\left[\frac{1}{p^2-6p+10}\right]$.

(8) Show that e^{t^2} is not of exponential order as $t \to \infty$.

2 (a) If S is a triangular region bounded by the line y=3x, x-axis and the line x=6, then find the value of $\iint_S x^2y^2 dx\,dy$. **5**

OR

(a) Evaluate $\iint_S (6-x-y)dx\,dy,$

where $S=\{(x,y) \mid x \ge 0, y \ge 0, x+y \le 6\}$

(b) Attempt any two : **10**

(1) Evaluate $\int_1^2\int_2^3 \left(y^2+2xy\right)dxdy$.

(2) Find the area between two parabolas $y^2=2x$ and $x^2=2y$.

(3) Change the order of integration of the double integral $\int_0^4\int_0^{\frac{y}{2}} f(x,y)dydx$.

(4) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} xy\,dx\,dy$ after changing the order of integration.

3 (a) In usual notation prove that $B[m,n]=\frac{\Gamma m \Gamma n}{\Gamma(m+n)}$. **10**

OR

(a) Define gamma integral and by proper substitutions, prove that $\Gamma n=\frac{1}{n}\int_0^{\infty} e^{-y^{\frac{1}{n}}} dy$.

(b) Attempt any two : **10**

(1) Evaluate $\int_0^{\infty} x^6 e^{-2x} dx$.

(2) Prove that $\int_0^1 x^{n-1}\left[\log\left(\frac{1}{x}\right)\right]^{m-1} dx=\frac{\Gamma m}{n^m}(m,n>0)$.

(3) Evaluate $\int_0^2 \frac{x^2}{\sqrt{2-x}} dx$.

(4) Show that $\int_0^{\infty} \cos\left(b x^{\frac{1}{n}}\right) dx=\frac{\Gamma(n+1)}{b^n}\cos\left(\frac{n\pi}{2}\right)(n>0)$.

4 (a) If $L[F(t)]=f(p)$, where $p>\alpha$, then **5**

show that $L\left[e^{at}F(t)\right]=f(p-a)$

where $p>a+\alpha$. Using this property find $L\left[e^{at}\sin 3t\right]$.

OR

(a) Show that the Laplace transformation is linear. Obtain $L[\cosh at]$.

(b) Find Laplace transformation of the following. (any two) **10**

(1) $\sin\sqrt{t}$

(2) t^5 (Using the definition of Laplace transformation)

(3) $F(t)=\begin{cases}\sin t, & 0<t<\pi\\ 0, & t<\pi\end{cases}$

(4) $F(t)=\begin{cases}Cos\left(t-\frac{2}{3}\pi\right), & t>\frac{2}{3}\pi\\ 0, & t<\frac{2}{3}\pi\end{cases}$

5 (a) In usual notation prove that **5**

$$L^{-1}[f(p)]=F(t)\Rightarrow L^{-1}[f(ap)]=\frac{1}{a}F\left(\frac{t}{a}\right).$$

OR

(a) State and prove second shifting theorem for inverse Laplace transform.

(b) Prove the following. (any two) **10**

(1) $L^{-1}\left[\frac{p}{\left(p^2+1\right)^2}\right]=\left(\frac{t}{2}\right)\sin t\Rightarrow L^{-1}\left[\frac{32p}{\left(16p^2+1\right)^2}\right]=\left(\frac{t}{4}\right)sin\left(\frac{t}{4}\right).$

(2) $L^{-1}\left[\frac{e^{-\pi p}}{p^2+1}\right]=\begin{cases}\sin(t-\pi), & t>\pi\\ 0, & t<\pi\end{cases}.$

(3) $L^{-1}\left[\frac{2p+3}{(p+1)(p+2)}\right]=e^{-1}+e^{-2t}.$

(4) $L^{-1}\left[\frac{3p-2}{p^2-4p+20}\right]=e^{2t}\left(3\cos 4t+\sin 4t\right).$
