* J B - 3 1 9 4 *

JB-3194

Second Year B. Sc. (Sem. - IV) Examination

April/May – 2013

Mathematics : CCM - 402

(New Course)

(Partial Differential Equations)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **S. Y. B. Sc. (Sem. - IV)**

Name of the Subject :

☛ **Mathematics : CCM - 402 (NEW COURSE)**

☛ Subject Code No. : **3 1 9 4** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Digits to the right of each question indicate its marks.

(4) Follow usual notations.

1 Attempt any five out of eight : **10**

(1) Write Lagrange's subsidiary equations for

$$\cos x \frac{\partial z}{\partial x} - \sec x \frac{\partial z}{\partial y} = e^{-2}$$

(2) Find the complete solution of $p^2 + q^2 = m^2$.

(3) Obtain complete solution of $q - p + x - y = 0$.

(4) Solve : $p + q = \frac{z}{a}$.

(5) Show that the solution of $p \cdot q = z$ *is* $z = (x+a)(y+b)$.

(6) Obtain the complementary function of $\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2} = x^2$.

(7) Obtain the particular integral of $\frac{\partial z}{\partial x}-6\frac{\partial z}{\partial y}=x^2$.

(8) Find : $\frac{1}{(D-D')}e^{x+y}$.

2 (a) Obtain the partial differential equation by eliminating arbitrary constants a, b and c from $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. **5**

OR

(a) Obtain the partial differential equation by eliminating arbitrary function ϕ from $\phi\left(x+y+z, x^2+y^2-z^2\right)=0$. **5**

(b) Solve any two out of four : **10**

(1) $xzp+yzq=x\cdot y$

(2) $\left(z^2-2yz-y^2\right)p+(xy+xz)q=xy-xz$

(3) $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$

(4) $pz-qz=z^2+(x+y)^2$

3 (a) Explain the method to solve F(p,q) = 0. **5**

OR

(a) Describe the method of solving $z=px+qy+f(p,q)$ **5**

(b) Solve any two out of four : **10**

(1) $p\left(q^2+1\right)+(a-z)q=0$

(2) $p^2+q^2=x+y$

(3) $z-px-qy=p^2+q^2$

(4) $z=p\cdot q$

4 (a) Describe the method of finding the complete solution of **5**

$\left(D^2+k_1DD'+k_2D'^2\right)z=0$,

where $D\equiv\frac{\partial}{\partial x}$ and $D'\equiv\frac{\partial}{\partial y}$;

k_1,k_2 are constants, and the roots of auxiliary equation of the given partial differential equation are real and distinct.

OR

(a) Show that the particular integral of **5**

$f\left(D^2,DD',D'^2\right)z=\sin(mx+ny)$ is given by

$$P.I=\frac{1}{f\left(-m^2,-mn,-n^2\right)}.\sin(mx+ny).$$

(b) Solve any two out of four : **10**

(1) $\left(2D^2+5DD'+2D'^2\right)z=0$

(2) $\left(D^2+6DD'+9D'^2\right)z=0$

(3) $\frac{\partial^2z}{\partial x^2}-\frac{\partial^2z}{\partial x\partial y}=\cos x\cdot\cos 2y$

(4) $\frac{\partial^2z}{\partial x^2}-4\frac{\partial^2z}{\partial x\partial y}+4\frac{\partial^2z}{\partial y^2}=e^{2x+y}$

5 (a) Define the non-homogeneous linear partial differential equation. Hence show that the complementary function of **5**

$f(D,D')z=f(x,y)$ is given by $C.F.=\phi_1(y+mx)+e^{cx}\phi_2(y+mx)$

where $f(D,D')\equiv(D-mD').(D-mD'-C)$.

OR

(a) Obtain Monge's equation from $R.r+S.s+T.t=V$. **5**

(b) Solve any two out of four : **10**

(1) $\left(D^2+2DD'+D'^2-2D-2D'\right)z=\sin(x+2y)$

(2) $\left(D^2+DD'+D'-1\right)z=e^{-x}$

(3) $(D+D'-1)\cdot(D+2D'-3)z=4+3x+6y$

(4) $\left(D^2+DD'+D'-1\right)z=\cos(x+2y)$.