



JB-3199
Second Year B. Sc. (Sem. IV) Examination
April/May – 2013
Mathematical Modelling (I.D.S.)
(New Course)

Time : 3 Hours]

[Total Marks : 70

Instructions :

(1)

<p>નીચે દર્શાવેલ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fillup strictly the details of signs on your answer book.</p> <p>Name of the Examination :</p> <p>← S. Y. B. Sc. (Sem. - IV)</p> <p>Name of the Subject :</p> <p>← Mathematical Modelling (I.D.S.) (NEW COURSE)</p> <p>← Subject Code No. : 3 1 9 9 ← Section No. (1, 2,.....) : Nil</p>	<p>Seat No. :</p> <table border="1" style="width: 100%; height: 20px;"><tr><td style="width: 15%;"></td><td style="width: 15%;"></td><td style="width: 15%;"></td><td style="width: 15%;"></td><td style="width: 15%;"></td><td style="width: 15%;"></td></tr></table> <div style="border: 1px solid black; border-radius: 15px; padding: 10px; text-align: center; margin-top: 10px;">Student's Signature</div>						

- (2) All questions are compulsory.
(3) Figures to the right indicate marks of the question.
(4) Follow usual notations.

1 Answer the following questions. 10

- (1) If Rs. 1000 is deposited at a compound interest rate 6% per year, find the amount deposited after six years, if the interest compounded monthly ?
- (2) State only Fick's law of diffusion.
- (3) State only Newton's cooling law.
- (4) Write only differential equation of SIS model with constant number of carriers.
- (5) In the change in price of commodity model $p_e - p(t) = (p_e - p(0))e^{-kt}$,. Discuss what happens if $t \rightarrow \infty$.

2 (a) Derive mathematical model for effect of immigration and imigration on population size and solve it. 8

OR

- (a) Derive mathematical model for interest compounded continuously and solve it. 8
- (b) In an archeological wooden specimen, only 25% of original radio carbon-12 is present. When was it made ? 7

OR

- (b) How much time does it take for a given amount of money to get double at 6% per annum compounded (1) annually (2) monthly (3) continuously ? 7

- 3 (a) Derive mathematical model for Fick's law of diffusion and solve it. 8

OR

- (a) Derive mathematical model for change in price of commodity and solve it. 8
- (b) The concentration of potassium in kidney is 0.0025 mg/cm^3 . The kidney is placed in a large Vessel in which potassium concentration is 0.0040 mg/cm^3 . In 1 hour the concentration in the kidney increases to 0.0027 mg/cm^3 . After how much time will the concentration be 0.0035 mg/cm^3 ? 7

OR

- (b) A body where temperature T is initially 300°C is placed in block of ice. Find its temperature at the end of 2 and 3 minutes ? 7

- 4 (a) Derive mathematical model for Newton's cooling law and solve it. 8

OR

- (a) Derive mathematical model of susceptible infected - Removal persons. 8
- (b) Integrate $\frac{dI}{dt} = \beta I(N+1-I)$, when $t=0$, 7

$I(0)=1$ then prove that $I(t) = \frac{(n+1)e^{(n+1)\beta t}}{n + e^{(n+1)\beta t}}$.

OR

(b) Integrate $\frac{dS}{dt} = -\beta S(n+1-S)$; when $t = 0$, 7

$S(0)=n$ then prove that $S(t) = \frac{n(n+1)}{n + e^{(n+1)\beta t}}$.

- 5 (a) Derive mathematical model for epidemic Susceptible - Infected persons. 8

OR

- (a) Derive mathematical model for Susceptible-Infected-Susceptible persons. 8

(b) Integrate $\frac{dI}{dt} = [\beta(n+1) - \gamma]I - \beta I^2$, when $t = 0$, 7

$I(0) = I_0$ then prove that

$$I(t) = \frac{(n\beta + \beta - \gamma)I_0 e^{(n\beta + \beta - \gamma)t}}{\left[(n\beta + \beta - \gamma) + \beta I_0 \left\{ e^{(n\beta + \beta - \gamma)t} - 1 \right\} \right]}$$

OR

(b) Integrate $\frac{dI}{dt} = \beta I [(n+1-\rho) - I]$, when $t=0$, 7

$I(0) = I_0$ then prove that

$$I(t) = \frac{(n+1-\rho)I_0 e^{(n+1-\rho)\beta t}}{\left[(n+1-\rho) + I_0 \left\{ e^{(n+1-\rho)\beta t} - 1 \right\} \right]}$$