* J B - 3 1 9 8 *

JB-3198

Second Year B. Sc. (Sem. IV) Examination

April/May – 2013

Mathematics : CCM - 403 (CS)

(Graph Theory)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **SECOND YEAR B. SC. (SEM. 4)**

Name of the Subject :

☛ **MATHEMATICS : CCM - 403 (CS)**

☛ Subject Code No. : **3 1 9 8** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All questions are compulsory.
(3) Figures to the right indicate full marks.
(4) Follow usual notations.
(5) Use of scientific non-programmable calculator is allowed.

1 Answer any **five** from the following questions : **10**

(i) Define : Simple graph and Connected graph.
(ii) Draw two non-isomorphic, simple planar graphs with 6 nodes and 9 edges.
(iii) Explain : Ring-sum of two graphs.
(iv) Draw an Euler graph which is not arbitrary traceable.
(v) Define with illustration : Hamiltonian Circuit.
(vi) Define incidence matrix of a digraph.
(vii) Can minimally connected graph have a circuit ? Justify your answer.
(viii) Sketch one binary tree with six pendant vertices and find its path length.

2 (a) Prove that a connected graph with n vertices and e edges has $e-n+2$ regions. **5**

OR

(a) Explain : Konigsberg bridge problem. **5**

(b) Attempt any **two** of the following : **10**

(i) If a graph has exactly two vertices of odd degree, then prove that there exist a path joining these two vertices.

(ii) Prove that a graph with n vertices, $n-1$ edges and no circuits is connected.

(iii) Determine the number of edges in a graph with 6 nodes, 2 of degree 4 and 4 of degree 2. Draw two such graphs, one simple, one not.

(iv) How many edges must be drawn in order to obtain a planar graph with 5 nodes that defines 7 regions ? Draw such a graph.

3 (a) Prove that a connected graph G has an Euler circuit if and only if all the nodes of G are of even degree. **5**

OR

(a) Prove that a connected graph with exactly 2 nodes of odd degree has an Euler Path. **5**

(b) Attempt any **two** of the following : **10**

(i) If the intersection of two paths is a disconnected graph, then prove that the union of the two paths has at least one circuit.

(ii) Draw all simple, connected graphs with 4 nodes and no proper circuits, and count the number of edges in each. Repeat the same for 5 nodes. Generalize your results as to the number of nodes in a simple, connected graph with n nodes and no proper circuits.

(iii) Define with an illustration :

(a) Euler Circuit

(b) Euler Path

(c) Hamiltonian Path.

(iv) Find an Euler Circuit and Euler Path in each of the following graphs or show that none exists.

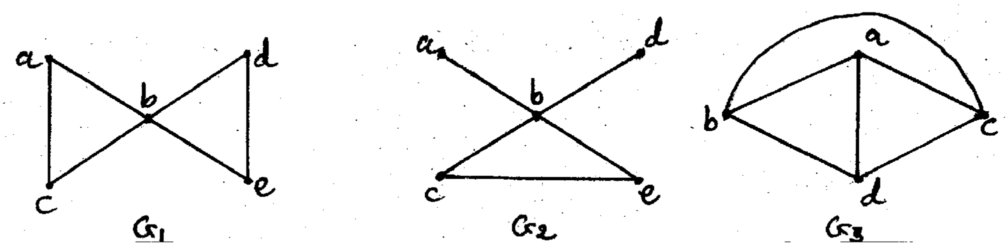

Fig.

4 (a) State and prove Euler's Theorem for digraphs. **5**

OR

(a) Define : **5**

(i) Simple Digraphs
(ii) Asymmetric Digraphs
(iii) Symmetric Digraphs
(iv) Simple Symmetric Digraphs
(v) Balanced Digraphs.

(b) Attempt any **two** of the following : **10**

(i) How many edges are there in a digraph with 5 nodes, each of which has out-degree 2 ? Draw such a digraph.
(ii) Let, $X = \{1, 2, 3, 4\}$ and $R = \{(x, y) \mid x > y\}$. Draw the digraph of R and also give its adjacency matrix.
(iii) Define : Incidence matrix, Adjacency matrix and Circuit matrix.
(iv) Draw a digraph of a binary relation R on A {1, 2, 3, 4, 5} defined by xR_y if y is divisible by x. Also characterize it in terms of equivalence relations.

5 (a) Prove that a tree with n vertices has n–1 edges. **5**

OR

(a) Prove that a graph is a tree iff it is minimally connected. **5**

(b) Attempt any **two** of the following : **10**

(i) Draw all rooted trees with 5 vertices.
(ii) Distinguish between a spanning tree and a minimal spanning tree. Also explain with graph.
(iii) Prove that the no. of leaves on a binary tree can be any number greater than 1.
(iv) Prove that every tree has either one or two centers.
