* J B - 3 1 9 5 *

JB-3195

Second Year B. Sc. (Sem. IV) Examination

April/May – 2013

Mathematics : Paper-CCM-403

(Numerical Analysis - II)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **S. Y. B. Sc. (Sem. 4)**

Name of the Subject :
☛ **Mathematics : Paper-CCM-403**

☛ Subject Code No. : **3 1 9 5** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) Figures to the right indicate marks of the question.

(3) Follow usual notations.

(4) Use of scientific non-programmable calculator is allowed.

1 Answer any **five** as directed : **10**

(1) Discuss the disadvantage of Lagrange's Interpolation formula and the advantage of Newton's Divided-Difference Interpolation formula.

(2) (i) State Lagrange's formula for 'Inverse Interpolation.'

(ii) Show that the divided-difference of order 1 is symmetric in their arguments.

(3) If $f(x)=\frac{1}{x^2}$, then find the divided-differnece [a,b].

(4) Find the value of the divided-difference $\left[x_0, x_0\right]$.

(5) If $x_1 - x_0 = x_2 - x_1 = h > 0$, then find the divided-difference $\left[x_0, x_1, x_2\right]$.

(6) Prepare the divided-differnece table for :

x	-1	0	3	6	7
y	3	-6	39	822	1611

(7) State the necessary conditions for applying Simpson's-$\frac{1}{3}$ Rule and Simpson's-$\frac{3}{8}$ Rule to evaluate the integral

$$I = \int_{x_0}^{x_n} y \cdot d\,x$$

(8) State the formula for

$$\left[\frac{dy}{dx}\right]_{x=x_n} \text{ and } \left[\frac{d^2y}{dx^2}\right]_{x=x_0}.$$

2 (a) Derive Lagrange's Interpolation formula. **5**

OR

(a) Derive Newton's Divided-Difference Interpolation formula. **5**

(b) Attempt any **two** : **10**

(i) Using Lagrange's Interpolation formula, find $f(0)$ for the table :

x	-1	1	2
y	-3	-1	3

(ii) Using Lagrange's Interpolation formula, find $f(x)$ for the table :

x	-1	0	3
y	3	-6	39

(iii) Using Newton's Divided-Difference Interpolation formula, find $f(0)$ for the table :

x	-1	1	2	3
$y = f(x)$	-21	15	12	3

(iv) Using Newton's Divided-Difference Interpolation formula, find $f(x)$ for the table :

x	1	3	4	6
$y = f(x)$	-3	9	30	132

3 (a) Prove in usual notations that : **5**

$$\left[\frac{dy}{dx}\right]_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \ldots\right]$$

OR

(a) Prove in usual notations that : **5**

$$\frac{d^2y}{dx^2}=\frac{1}{h^2}\left[\nabla^2 y_n+(p+1)\nabla^3 y_n+\frac{6p^2+18p+11}{12}\nabla^4 y_n+\ldots\right]$$

(b) Attempt any **two** : **10**

(1) Find $\frac{dy}{dx}$ at $x=1$, for the tatble :

x	0	1	2	3	4	5
y	0	3	7	15	38	50

(2) Find $\frac{d^2y}{dx^2}$ at $x=1.5$, for the table :

x	1.5	2.0	2.5	3.0	3.5
y	3	9	18	28	40

(3) Find $\frac{dy}{dx}$ at $x=1.6$, for the table :

x	1.0	1.2	1.4	1.6	1.8
y	2.7183	3.3201	4.0552	4.9530	6.0496

(4) Find $\frac{d^2y}{dx^2}$ at $x=0.4$, for the table :

x	0.0	0.1	0.2	0.3	0.4
y	1.0000	0.9975	0.9900	0.9776	0.9604

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4 (a) Derive the general formula for integration, using Newton's Forward-Diference Interpolation formula. **5**

OR

(a) State and prove Simpon's $\frac{1}{3}$ Rule. **5**

(b) Attempt any **two** : **10**

(1) Evaluate $I=\int_1^3 \frac{1}{x}dx$, using Trapezoidal Rule, by dividing the interval $[1,3]$ into 8 (eight) equal subintervals.

(2) Evaluate $I=\int_0^1 \frac{1}{1+x}dx$, using Simpon's-$\frac{1}{3}$ Rule, by taking $h=\frac{1}{6}$.

(3) Evaluate $I=\int_0^1 \frac{1}{1+x^2}\cdot dx$, using Simpon's-$\frac{3}{8}$ Rule, by taking $h=\frac{1}{6}$.

(4) Evaluate $I=\int_0^1 \sqrt{1-x^2}dx$, by using any Rule, taking $h=\frac{1}{6}$.

5 (a) Explain Euler's method to solve the Initial-Value problem : 5

$\frac{dy}{dx}=f(x,y)$; $y(x_0)=y_0$.

OR

(a) Explain Picard's Approximation method to solve the Initial - value problem : 5

$\frac{dy}{dx}=f(x,y)$; $y(x_0)=y_0$.

(b) Attempt any **two** :

(1) Obtain the Taylor's series for $y(x)$, for the Initial-value problem :

$\frac{dy}{dx}=1+xy$; $y(0)=1$.

(2) Using Euler's method, for the Initial-value problem :

$\frac{dy}{dx}=-y$; $y(0)=1$, find $y(0.01)$, $y(0.02)$ and $y(0.03)$, by taking h=0.01.

(3) Using Picard's approximation method, solve the Initial-value problem :

$\frac{dy}{dx}=1+y^2$; $y(0)=0$, upto three (3) qpproximations.

(4) Using Taylor's series method find $y(0.2)$, for the Initial-value problem :

$\frac{dy}{dx}=x+y$; $y(0)=0$.