* J B - 3 2 0 1 *

JB-3201

Second Year B. Sc. (Sem. IV) Examination

April/May – 2013

Mathematical Methods - II

(IDS)

Time : 3 Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **S. Y. B. Sc. (Sem. 4)**

Name of the Subject :
☛ **Mathematical Methods - 2**

☛ Subject Code No. : **3 2 0 1** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) First question is **compulsory**.
(3) Figures to the right indicate marks of question.
(4) Follow usual notations.

1 Answer the following questions : **10**

(1) IF $\alpha + i\beta$ is root of the quadratic equation $f(x) = 0$ then express $f(x)$ as the product of two linear factors.

(2) If α, β, γ rate the roots of equation $f(x) = 0$ then write the equation whose roots are $\frac{-1}{\alpha}, \frac{-1}{\beta}, \frac{-1}{\gamma}$.

(3) State all the three cube roots of unity. Also state the real and imaginary parts of its complex roots.

(4) For any complete polynomial of n[th] degree, if the number of change in signs is λ and number of continuation is μ then state the value of $\mu + \lambda$.

(5) Find the number of change in signs for the polynomial $x^7 + 2x^6 - 3x^5 - 4x^4 - 2x^3 + 5x^2 - 6x + 11 \cdot$

2 (a) If the polynomial $x^n + p_1 x^{n-1} + p_2 x^{n-2} + \cdots + p_n = 0$ has roots $\alpha_1, \alpha_2, \ldots, \alpha_n$ then derive the relation between these roots and coefficients of the polynomial. **5**

OR

(a) State the fundamental theorem of Theory of Equations. Hence show that any n^{th} degree polynomial equation has n roots. **5**

(b) Attempt any **two** of the following : **10**

(1) If α, β, γ are the roots of the equation $2x^3+3x^2-x-1=0$ then find the equation whose roots are $2\alpha+3$, $2\beta+3$, $2\gamma+3$.

(2) If α, β, γ are the roots of the equation $x^3-x-1=0$ then find the equation whose roots are $\frac{1+\alpha}{1-\alpha}$, $\frac{1+\beta}{1-\beta}$, $\frac{1+\gamma}{1-\gamma}$.

(3) If α, β, γ are the roots of the equation $8x^3-4x^2+6x-1=0$ then find the equation whose roots are $\alpha+\frac{1}{2}$, $\beta+\frac{1}{2}$, $\gamma+\frac{1}{2}$.

(4) Solve the equation $4x^4-4x^3-25x^2+x+6=0$ when the difference between its two roots is 1.

3 (a) State and prove the Descarte's rule of signs. **5**

OR

(a) Convert the equation $f(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\cdots+a_n=0$ into an equation whose coefficient in the leading term is 1 and other terms have minimum coefficients. **5**

(b) Attempt any **two** of the following : **10**

(1) Convert the equation $x^3-6x^2+4x-7=0$ into an equation which does not contain the second term.

(2) From the equation $72x^3-54x^2+45x-7=0$, obtain an equation whose coefficient of the term having x^3 is 1 and other terms have minimum coefficients.

(3) Show that the equation $x^4+7x^2+2x-9=0$ has one positive, one negative and one pair of complex roots.

(4) Using Descarte's rule of signs show that $x^7-3x^4+2x^3-1=0$ has at least four complex roots.

4 Attempt any **two** of the following : **15**

(1) Using Cardan's method, obtain real root of the equation

$x^3 + 29x - 97 = 0$.

(2) Solve the equation $x^3 + 9x - 6 = 0$ using Cardan's method.

(3) If α, β, γ are the roots of the equation

$ax^3 + 3bx^2 + 3cx + d = 0$ and if $H = ac - b^2$,

$G = a^2d - 3abc + 2b^3$ then prove that

$a^3(2\alpha - \beta - \gamma)(2\beta - \gamma - \alpha)(2\gamma - \alpha - \beta) = -27G$.

(4) Solve $x^4 - 4x^3 + 2x^2 + 4x - 3 = 0$ by Ferrari's method.

5 Solve any two of the following differential equation by the method of variation of parameters : **15**

(1) $\frac{d^2y}{dx^2} + n^2y = \sec nx$

(2) $y'' + y = \operatorname{cosec} x$

(3) $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$

(4) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}\cos x$.