* J - 0 8 5 1 *

J-0851

Second Year B. Sc. (C.S.) Examination

March/April – 2013

Mathematics - IV

(Discrete Mathematics)

(Old Course)

Time : **3** Hours] [Total Marks : **105**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **SECOND YEAR B. SC. (C.S.)**

Name of the Subject :
☛ **MATHEMATICS - 4 (OLD)**

☛ Subject Code No. : **0 8 5 1** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate full marks.

1 Answer the following questions : **15**

(1) Define well ordered set with illustration.

(2) State distributive law for lattice.

(3) Define equivalence relation with illustrations.

(4) Define :

(a) Hamiltonian path

(b) Component of graph

(c) Regular graph.

(5) Define Modular lattice.

2 (a) Let R be the relation an N defined by xRy if x and y sharena common factor other then 1; determine the reflectivity and transitivity of R. **18**

(b) Define diagraph. Let $S = \{2, 3, 4, 6, 8, 9, 12\}$ and let the relation S be define by xSy if x divides y then draw the diagraph.

(c) Let $\langle L, \leq \rangle$ be a lattice, prove that $\forall a, b, c \in L$

$$b \leq c \Rightarrow \begin{cases} a * b \leq a * c \\ a \oplus b \leq a \oplus c \end{cases}$$

OR

2 (a) Let R be a binary relation an A, then prove that transitive closure $R, t(R) = \bigcup_{i=1}^{\infty} R^i$. **18**

(b) Solve :

(i) $x - 3 \equiv 2 (\text{mod } 7)$

(ii) $x \equiv -1 (\text{mod } 2)$

(c) Show that relation R_5 is an equivalence relation.

3 (a) Prove that the component of an element of Boolean algebra is unique. **18**

(b) Using the rules of Boolean algebra prove that $x_1 x_2 + x_1 x_2' + x_1' x_2 + x_1' x_2' = 1$.

(c) Obtain the product of sum canonical form of the boolean expressions :

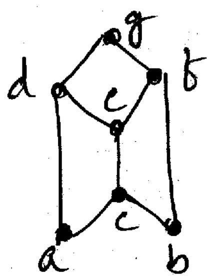
(i) $x_1 * x_2$

(ii) $x_1 \oplus x_2$

OR

3 (a) In a Boolean algebra prove that $a = b \Leftrightarrow ab' + a'b = 0$. **18**

(b) Using the given hasse-diagram, find lub and glb of $\{a, b\}$, $\{a, c, f\}$, $\{d, e, f\}$, $\{g, d, f\}$.



(c) Prove that (mod m) relation is equivalence relation.

4 (a) Show that in a lattice if $a \leq b \leq c$ then **18**

(i) $a \oplus c = b * c$

(ii) $(a * b) \oplus (b * c) = b = (a \oplus b) * (a \oplus c)$

(b) 'Every poset is not lattice' explain with illustration.

(c) Prove that every chain is a distributive lattice.

OR

4 (a) Define : **18**

(i) Complete graph

(ii) Hamiltonian circuit

(iii) Pendent vertex

(iv) Adjacent vertices.

(b) Explain Konigsberg Bridge problem.

(c) Prove that the no. of vertices of odd degree in a graph is always even.

5 (a) Prove that the sum of degrees of all vertices in any graph is twice the number of edges in it. **18**

(b) Justify the statements :

(i) Energy regular graph is a Euler graph.

OR

(ii) Every Euler graph is a regular graph.

(c) Discuss Utility problem.

OR

5 (a) A given connected graph G has a Euler circuit iff all the vertices of G are of even degree. **18**

(b) Discuss Sitting problem.

(c) Define Finite and Infinite graph and prove that an infinite graph with finite no. of edges can have infinite no. of isolated vertices.

6 (a) Determine the number of regions defined by a connected plannar graph with 4-vertices and 8-edges. Draw such graph. **18**

(b) Draw arithmetic tree :

(i) $\{(1+2)*[(3-4)+(1-5)]\}-[(6-3)-8]$

(ii) $5*(7+(3-4)**3)$

(c) Prove that a graph is a tree iff it is minimally connected.

OR

6 (a) Draw all rooted tree with 5-vertices. **18**

(b) Use k-map to find minimal sum of the following :

(i) $xyz + xyz' + x'yz' + x'y'z$

(ii) $xy' + xyz + x'y'z' + x'yz'$

(c) Explain Prim's Algorithm to obtain spanning tree with illustration.