* J - 0 8 4 8 *

J-0848

Second Year B. Sc. Examination

March/April – 2013

Mathematics : Paper - IV

(Old Course)

Time : Hours] [Total Marks :

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **Second Year B. Sc.**

Name of the Subject :
☛ **Mathematics : Paper - 4 (Old)**

☛ Subject Code No. : **0 8 4 8** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate marks of the corresponding question.

1 Do as directed. **15**

(1) Obtain general solution of $\frac{d^2y}{dx^2} - y = \left(e^x + 1\right)^2$.

(2) Obtain general solution of $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = 0$.

(3) Obtain $L\{1\}$ and $L\{e^{at}\}$.

(4) Eliminate h and k from the equation

$(x-h)^2 + (y-k)^2 + z^2 = c^2$.

(5) Obtain $L^{-1}\left\{\frac{2p+1}{p(p+1)}\right\}$ and $L^{-1}\left\{\frac{1}{p^2 - 6p + 10}\right\}$.

2 (a) In usual notations prove that **6**

$\frac{1}{f(D)}(xV) = \left\{x - \frac{1}{f(D)}f'(D)\right\}\frac{1}{f(D)}V$; where V is a function of x.

(b) Solve : **12**

(i) $\frac{d^2y}{dx^2}+2y=x^2e^{3x}+e^x\cos 2x$.

(ii) $\frac{d^3y}{dx^3}+8y=x^4+1$.

OR

2 (a) In usual notations prove that $\frac{1}{f(D)}e^{ax}V=e^{ax}\frac{1}{f(D+a)}V$. **6**

(b) Solve : **12**

(i) $\frac{d^3y}{dx^3}+y=\sin 3x-\cos^2\frac{x}{2}$.

(ii) $\frac{d^2y}{dx^2}-y=x^2\cos x$.

3 (a) Write Legendre's differential equation and explain the method to solve it. **6**

(b) Solve : **12**

(i) $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^4$.

(ii) $x^3\frac{d^3y}{dx^3}+x\frac{dy}{dx}-y=3x^4$.

OR

3 (a) In usual notations prove that $\frac{1}{f(\theta)}x^m=\frac{1}{f(m)}x^m$; **6**

$f(m)\neq 0$.

(b) Solve : **12**

(i) $(5+2x)^2\frac{d^2y}{dx^2}-6(5+2x)\frac{dy}{dx}+8y=2(5+2x)^2$.

(ii) $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x$.

4 (a) Discuss how to solve linear differential equation of second order by changing the independent variable. **6**

(b) Solve : **12**

(i) $\frac{d^2y}{dx^2}+\frac{2}{x}\frac{dy}{dx}+\frac{a^2}{x^4}y=0$.

(ii) $\frac{d^2y}{dx^2}-\cot x\frac{dy}{dx}+y\sin^2 x=0$.

OR

4 (a) When one integral of linear differential equation of second order is known to us then discuss how to obtain second integral of it. **6**

(b) Solve : **12**

(i) $\frac{d^2y}{dx^2}-\frac{1}{x}\frac{dy}{dx}+4x^2y=x^4$.

(ii) $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+\left(4x^2-1\right)y=-3e^{x^2}$.

5 (a) State and prove the change of scale property for Laplace transform. **6**

(b) Evaluate $L\{2t3+4e^{-3t}+3\cos 2t\}$. **6**

(c) Using Laplace transform obtain the solution of **6**

$$\frac{d^2y}{dt^2}+m^2x=a\cos nt,\ t>0;\ x(0)=x_0,\ x'(0)=x_1,\ n\neq m.$$

OR

5 (a) State and prove the second shifting theorem for inverse Laplace transform. **6**

(b) Evaluate : $L^{-1}\left\{\frac{3p-2}{(p-2)^2+16}\right\}$ **6**

(c) Evaluate : $L\left\{e^{-4t}\cosh 2t\right\}$. **6**

6 (a) Explain the method to solve partial differential equation of the form $F(p,q)=0$. **6**

(b) Solve : **12**

(i) $q=-xp+p^2$

(ii) $q-p+x-y=0$.

OR

6 (a) Explain the method to solve partial differntial equation of the form $F(z,p,q)=0$. **6**

(b) Solve : **12**

(i) $p^2+q^2=n^2$

(ii) $p^2+q^2=x+y$.