J-0849

Second Year B. Sc. Examination

March/April – 2013

Mathematics : Paper - V

Time : 3 Hours] [Total Marks : 70

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ S. Y. B. Sc.

Name of the Subject :
☛ Mathematics : Paper - 5

☛ Subject Code No. : 0 8 4 9 ☛ Section No. (1, 2,.....) : Nil

Seat No. :

Student's Signature

(2) First question is **compulsory**.

(3) Figures to the right indicate marks of question.

(4) Follow usual notations.

1 Answer the following questions : **10**

(1) Define : Closure of a set. Is $f:(x,y)\to|x-y|$ a binary operation on N ? Why?

(2) In any vector space V, prove that $\alpha\theta=\theta$; for any scalar α.

(3) The set $\{(1,0,0),(1,1,1),(1,2,3)\}$ is LI or LD in V_3 ?

(4) Define : Norm of a vector in an inner product space *V*. Prove that $\|\alpha u\|=|\alpha|\|u\|; u\in V$ and α is scalar.

(5) Define : Unitary Matrix. Show that $\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ is unitary matrix.

2 (a) Let *S* be a non-empty subset of a vector space *V*. If $u+v\in S$ and $\alpha u\in S$; for $u,v\in S$ and scalar α, then prove that *S* is a subspace of *V*. **5**

(b) If the operations of addition and scalar multiplication are defined on R^+ by $u+v=u\cdot v$ and $\alpha u=u^{\alpha}$ (for all $u,v\in R^+$ and real scalar α), then prove that R^+ is a real vector space. **4**

(c) In a vector space V, if the set $\{v_1,v_2,....v_n\}$ is LI and $v\notin[v_1, v_2, ..., v_n]$, then prove that the set $\{v,v_1,v_2,....v_n\}$ is LI. **3**

OR

2 (a) If S is non-empty subset of a vector space V, then show that $[S]$ is the smallest subspace of V containing S. **5**

(b) Show that the set of all real valued functions defined on [0,1] is a group under pointwise addition of functions. **4**

(c) Define : LD set of vectors. If a set is LD, then show that any superset of it is also LD. **3**

3 (a) In an n-dimensional vector space V, prove that any set of n LI vectors is a basis for V. **5**

(b) Show that the ordered set {(1,1,0),(0,1,1),(1,0,-1),(1,1,1)} is LD and locate one of the vectors that belongs to the span of the previous ones. **4**

(c) In a vector space V, prove that the set $\{v_1,v_2,v_3\}$ is LD iff v_1,v_2 and v_3 are coplanar. **3**

OR

3 (a) If U and W are subspaces of a finite dimensional vector space V, then prove that **7**

$$\dim(U+W)=\dim U+\dim W-\dim(U\cap W).$$

(b) Show that the ordered set **5**

$A=\{(1,1,0),(0,1,1),(1,0,-1),(1,1,1)\}$ is LD and locate one of the vectors that belongs to the span of the previous ones. Hence find the largest LI subset whose span is [A].

4 (a) State and prove Rank-Nullity Theorem for a linear transformation $T:U \to V$. **7**

(b) Let $T:U \to V$ be a linear transformation. Prove that **5**

(1) $R(T)$ is a subspace of V.

(2) $N(T)$ is a subspace of U.

OR

4 (a) When two vector spaces U and V are isomorphic ? Prove that every real vector space of dimension p is isomorphic to V_p. **7**

(b) Prove that the linear transformation $T:V_3 \to V_3$ defined by $T(e_1) = e_1 + e_2, T(e_2) = e_2 + e_3, T(e_3) = e_1 + e_2 + e_3$ is nonsingular and find its inverse. **5**

5 (a) Prove that every real vector space of dimensions p is isomorphic to V_p. **7**

(b) Find range, kernel, rank and nullity for the matrix **5**

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

OR

5 (a) Let $T:U \to V$ and $S:V \to W$ be two nonsingular linear transformations. Prove that **7**

(1) ST is nonsingular and $(ST)^{-1} = T^{-1}S^{-1}$.

(2) If ST is one-one, then T is one-one.

(b) Let $T = V_3 \to V_2$ be defined by $T(x, y, z) = (x + y, y + z)$. **5**

If $B_1 = \left\{\left(1,1,\frac{2}{3}\right),(-1,2,-1),\left(2,3,\frac{1}{2}\right)\right\} and\, B_2 = \left\{(1,3)\left(\frac{1}{2},1\right)\right\}$

then determine $(T:B_1,B_2)$.

6 (a) State and prove Schwarz's inequality for the inner porudct space V. **5**

(b) Orthonormalize the LI set $\{(1,2,1),(-1,1,0),(5,-1,2)\}$ by Gram-Schmidt process. **7**

OR

6 (a) Prove that any orthogonal set of nonzero vectors in an inner product space is LI. **5**

(b) Consider the vector space $\mathcal{C}[0,1]$. Define **7**

$f \cdot g = \int_0^1 f(t)g(t)dt,\ where\ f,g \in \mathcal{C}[0,1]$. Prove that $\mathcal{C}[0,1]$ is an inner product space.