J-0850

Second Year B. Sc. Examination

March/April – 2013

Mathematics - III

(Diff. Calculus & Diff. Equations)

(Old Course) (Comp. Science)

Time : 3 Hours] [Total Marks : 105

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **SECOND YEAR B. SC.**

Name of the Subject :
☛ **MATHEMATICS - 3 (OLD)**

☛ Subject Code No. : **0 8 5 0** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

(2) Figures to the right indicate full marks of the questions.
(3) All the question are compulsory.

1 Answer the following questions : **15**

(1) Find the limit of

$$f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}} \quad ; \ (x, y) \neq (0, 0)$$

$$= 0 \quad ; \ (x, y) = (0,0) \text{ at the point } (0,0).$$

(2) Varify the function for its homogenity

$$f(x, y, z) = \frac{4x^3 + 2y^2 z}{x + 2y + 3z}$$

(3) Evaluate $\int_0^\infty x^4 e^{-x} dx$

(4) Find complementary function of $\left(D^3 + 2D^2 + 4D + 8\right) y = x^3$.

(5) If $x = r\cos\theta$, $y = r\sin\theta$ then find $\frac{\partial(x, y)}{\partial(r, \theta)}$

2 (a) If $f(D^2)$ is a polynomial in D^2 with constant co-efficients and $f(-a^2) \neq 0$ then prove that

(i) $\frac{1}{f(D^2)} \cos ax = \frac{1}{f(-a^2)} \cos ax$

(ii) $\frac{1}{f(D^2)} \sin ax = \frac{1}{f(-a^2)} \sin ax$

(b) Solve $(D^3 - 13D + 12)y = x$

(c) Solve $(D^4 - a^4)y = x^4$.

OR

2 (a) Prove that $\frac{1}{(D-a)^r} e^{ax} = \frac{x^r e^{ax}}{r!}$; $\forall r \in N$.

(b) Solve $(D^2 + 1)y = e^{2x} \cdot x$.

(c) Solve $(D^2 + 3D + 3)y = x^2 + 4x$.

3 (a) Show that a homogeneous linear differential equation can be transformed in to a linear differential equation with constant co-efficients by changing the independent variable from x to y by taking $z = \log x$.

(b) Solve $(D^2 + 3D + 4)y = x^2$

(c) Evaluate $\lim_{(x, y) \to (0, 0)} x^2 \sin\left(\frac{y}{x}\right)$

OR

3 (a) Prove that f is a differentiable homogeneous function of two variable x and y of degree $m \Leftrightarrow x\, f(x) + y\, f(y) = m\, f(x, y)$.

(b) Verify Eulers theorem for $u = \tan^{-1}\left(y/x\right) + \tan^{-1}\left(x/y\right)$,

(c) Find the extreme values of $x^3 y^2 (1 - x - y)$.

4 (a) Define Gamma function and prove that **18**

$$\frac{\Gamma n}{k^n} = \int_0^\infty e^{-ky} \cdot y^{n-1} dy.$$

(b) Prove that $\int_0^\infty \cos^2 x dx = \frac{1}{2}\sqrt{\pi/2}$

(c) Prove that $\int_0^\infty \frac{x^4\left(1+x^5\right)}{(1+x)^{15}} dx = \frac{1}{5005}$.

OR

4 (a) Define Beta function and prove that **18**

$\beta(m, n) = \beta(m+1, n) + \beta(m, n+1)$

(b) Find the extreme value of $f(x, y) = x^3 + y^3 - 3x - 12y + 5$.

(c) Prove that $\int_0^1 \frac{x}{\sqrt{1-x^5}} dx = 1/5\, \beta\left(2/5, 1/2\right)$.

5 (a) In usual notation prove that

$$\frac{1}{f(D)} x \cdot V = \left\{x - \frac{1}{f(D)} \cdot f'(D)\right\} \frac{1}{f(D)} \cdot V$$

where V is a function of x.

(b) Solve $x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$.

(c) If $u_1 \frac{x_2 x_3}{x_1}$, $u_2 = \frac{x_1 x_3}{x_2}$, $u_3 = \frac{x_1 x_2}{x_3}$ then prove that

$J(u_1, u_2, u_3) = 4$.

OR

5 (a) Evaluate $\iint_S xy\,dx\,dy$ where S is the region bounded by $y = 0$, $x = 2$ and $x^2 = 4y$.

(b) Solve $d^3y/dx^3 + y = e^{x/2} \cdot \sin\left(\sqrt{3}/2\, x\right)$.

(c) Solve $(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$.

6 (a) Solve using Gauss elimination method $2x + 2y + 4z = 18$, $x + 3y + 2z = 13$, $3x + y + 3z = 14$.

(b) Solve using Gauss-Seidel method.
$10x + y + z = 12$, $2x + 10y + z = 13$, $2x + 2y + 10z = 14$

(c) Evaluate $\iint_S (6 - x - y)\,dxdy$ where S is bdd. by $x \geq 0$, $y \geq 0$, $x + y \leq 6$.

OR

6 (a) Solve using Gauss Elimination method
$3x + y + 2z = 3$, $2x - 3y - z = 3$, $x + 2y + z = 4$

(b) Solve using Gauss Seidel method
$2x - 3y + 4z = 7$, $5x - 2y + 2z = 7$, $6x - 3y + 10z = 23$

(c) Change the order of integration of double integral

$$\int_0^{2\cos\theta} \int_{x\tan\theta}^{\sqrt{r^2 - x^2}} f(x, y)\,dx\,dy$$