J-0847

Second Year B. Sc. Examination

March/April – 2013

Mathematics : Paper - III

(Old Course)

Time : Hours] [Total Marks : **105**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી. **Fillup strictly the details of ☛ signs on your answer book.**	Seat No. :
Name of the Examination : ☛ **Second Year B. Sc.**	
Name of the Subject : ☛ **Mathematics : Paper - 3 (Old)**	
☛ Subject Code No. : **0 8 4 7** ☛ Section No. (1, 2,.....) : **Nil**	Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate marks of the corresponding question.

1 Do as directed. **15**

(1) Obtain $\lim_{x \to 0} \left\{ \lim_{y \to 0} \frac{x^2 - y^2}{x^2 + y^2} \right\}$ and $\lim_{y \to 0} \left\{ \lim_{x \to 0} \frac{x^2 - y^2}{x^2 + y^2} \right\}$.

(2) Evaluate $\int_0^1 x^4 (1-x)^3 \, dx$.

(3) If $f(x, y) = \sqrt{xy} + \tan^{-1}(y/x)$, $x \neq 0$, then find f_x and f_y.

(4) Evaluate $\int_1^2 \int_2^3 \left(y^2 + 2xy\right) dy dx$.

(5) Define countable set, bounded set and bounded sequence.

2 (a) If $f(x,y)$ is a homogeneous function in x and y of degree m and if second order partial derivatives of f exists then prove that **6**

$$x^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} = m(m-1) f(x, y).$$

(b) Discuss the continuity of $f(x,y)$ at the point $(0,0)$ **6**

where $f(x,y) = \dfrac{\sin(x+y)}{x+y}, \quad x+y \neq 0$

$= 0, \quad x+y=0.$

(c) Verify Euler's theorem for the function **6**

$f(x,y) = \tan^{-1}\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}.$

OR

2 (a) State and prove Euler's theorem for homogeneous function. **6**

(b) For $f(x,y) = \log\left(\sqrt{x^2+y^2}\right) + \tan^{-1}(y/x)$; examine whether f_{xy} and f_{yx} are equal or not. **6**

(c) If $H = f(y-z, z-x, x-y)$ then prove that **6**

$\dfrac{\partial H}{\partial x} + \dfrac{\partial H}{\partial y} + \dfrac{\partial H}{\partial z} = 0.$

3 (a) Obtain the expansion of $f(x,y) = \sin x \sin y$ in the form of powers of x and y. **6**

(b) If $x+y+z=u$, $y+z=uv$, $z=uvw$ then obtain $J(x,y,z)$ with respect to u, v and w. **6**

(c) Find the interior point of a triangle such that the sum of the squares of its distances from the vertices is minimum. **6**

OR

3 (a) Expand $f(x,y) = e^{ax}\cos by$ in the powers of x and y. **6**

(b) If it is given that $y_1(x_1 - x_2) = 0$, $y_2\left(x_1^2 + x_1x_2 + x_2^2\right) = 0$ **6**

then find $\dfrac{\partial(y_1, y_2)}{\partial(x_1, x_2)}$.

(c) Find the extreme values of $u = x^3 + y^3 - 3axy$. **6**

4 (a) If S is a triangular region bounded by the lines $y = 3x$, x-axis and the line $x = 6$, then find the value of $\iint_S x^2 y^2 \, dxdy$. **6**

(b) Evaluate $\int_0^1 \int_y^{2-y} (x^2, y^2) dxdy$ after changing the order of integration. **6**

(c) Show that $B(m,n) = B(m+1,n) + B(m,n+1)$. **6**

OR

4 (a) Find the area enclosed between two parabolas $y^2 = 2x$ and $x^2 = 2y$. **6**

(b) Show that $\lceil n+1 = n \lceil n$, hence for $n \in N$, prove that $\lceil n+1 = n!$. **6**

(c) Show that $\beta(1,m) = 2\int_0^{\frac{\pi}{2}} \sin^{21-1} \cos^{2m-1} d\theta$. **6**

5 (a) Show that the set of all rational numbers is countable. **6**

(b) Define a convergent sequence of a real numbers. If $\{S_n\} \to L \in R$ then show that L is unique. **6**

(c) Find $N \in 1^+ \ni \frac{1}{\sqrt{n+3}} < 0.03$, $\forall n \geq N$. **6**

OR

5 (a) Show that *[0,1]* is uncountable. **6**

(b) If $f : A \to B$ and range of f is uncountable then show that domain of f is uncountable. **6**

(c) Find $N \in 1^+ \ni \left|\frac{2n}{n+3} - 2\right| < \frac{1}{5}$, $\forall n \geq N$. **6**

6 (a) If $0 < x < 1$ then show that $\{x^n\}_{n=1}^{\alpha}$ converges to *0*. **6**

(b) Define Cauchy sequence and bounded sequence, also prove that a Cauchy sequence of real number is bounded. **6**

(c) State and prove Nested interval theorem. **6**

OR

6 (a) Prove that the non-increasing sequence of real numbers which is bounded below is convergent. **6**

(b) If $\{S_n\}_{n=1}^{\alpha}$ and $\{t_n\}_{n=1}^{\alpha}$ are sequences of real numbers and if $s_n = L$ and $\lim_{n\to\alpha} t_n = M$, where $L, M \in R$ then show that $\lim_{n\to\alpha}(s_n t_n) = LM$. **6**

(c) Find the limit superior and limit inferior for the following sequences, **6**

(i) $\left\{\sin\left(\frac{n\pi}{2}\right)\right\}_{n=1}^{\alpha}$

(ii) $\left\{\left(1+\frac{1}{n}\right)\cos n\pi\right\}_{n=1}^{\alpha}$.